(4)

Extraction of Roots
Geomet. Resol. Fra. van
Schooten
Annotations out of Wallis
Arith. Infinit.
Of Refractions
To Grind Elliptical
Lenses
To find the errors in Gravity
Series to rectify the
the annuity value
July 4th 1699
Note abt

Sep. 25 1727
Not fit to be printed
Tho. Pellet

See letter from
Professor H. W. Turnbull
in Add. 7339

H

Of ye extraction of ye Square Cubick. Square-square & square-cubick rootes &c.

Let ye number whose roote is to bee extracted bee pointed ~~bee~~ makeing ye first point under ye unite & comprizeing soe many numbers under each point as ye number hath dimensions as if ye number be square-cube tis thus pointed 5708 63524 10802.

Then ~~then then~~ out of ye figures of ye first point next ye left hand extract ye greatest roote proper to ye power of ye number & set yt downe in ye Quotient wch is ye first side & is called A. (as ye roote quintuplicate of 5708 is (5), & (5) quintuplicate is 3125) yn ~~takeing~~ yt roote duely multiplied out of ye number (as 3125 out of 5708) wth ye rest of ye numbers to ye next point. seek ye second side wch is found by divideing yt number by another number made out of ye first side (wch is called ye Divisor) as this ~~second side of name~~ E. (thus by divideing 2583 63524 by 5Aqq $+10Ac+10Aq+5A$ after such a man: yt $5AqqE+10AcEq+10AqEc+5AEqq+Eqc$ may be contained in ye number ye product of yt division shall be E =

The extraction of y^e sqare roote

The square to be resolved	29	16	(54 The Product.
The square of y^e first side to	25		be taken away.
The rest of y^e sqare to be	4	16	resolved.
The divisor for finding y^e seacond	1	0	side. w^{ch} is y^e first side (doubled
The rectangle by 2 A & E	4	0	} to be substracted
The square of E		16	
The sume of y^e rectangles	4	16	to be subducted.
	0	00	The remainder.

The extraction of y^e cube roote

The cube to be resolved	157	464	(54
The cube to be subducted	125		whose roote is A = 5
The remainder for y^e finding	32	464	of E
The divisors for y^e finding of E y^e seacond side.	7	5	3 Aq
		15	3 A
The sume of y^e divisors.	7	65.	
Sollids to be substracted	30	0	3 AqE
	2	40	3 AEq
		64	Ec
	~~32~~	~~44~~	
The sume of those	32	464	sollids
The remainder	0	0000	

The extraction of y^e square square roote

The square-square	33	1776	(24
The square-squ: to be subduc:	16		= Aqq
Remainder.	17	1776	
Divisors for finding y^e seacond side E.	3	2	4 Ac
		24	6 Aq
		8	4 A
Theire sume	3	448.	
Squ-squares to be sub=ducted	12	8	4 AcE
	3	84	6 AqEq
		512	4 AEc
		256	Eqq
Theire sume	17	1776	

The Extraction of ye Square-Cube roote

The squ: cube to be resolved	79	62624	(24
Substract	32	Aqc	
Remainder	47	62624	
Divisors	8	0	5Aqq
		80	10Ac
		40	10Aq
		10	5A
The Sume of ye divisors	8	8410	
Plano-sollids to be substracted	32	0	5AqqE
	12	80	10AcEq
	2	560	10AqEc
		2560	5AEqq
		1024	Eqc.
Their Sume	47	62624	
Remainder	00	00000	

Note yt ye 3d 4th 5th & other figures are found by ye same manner yt ye seacond figure is found Onely makeing all ye figures found to stand for A ye first side & ~~ye~~ ye figure sought for E or ye 2d side.

And if ~~ye number propounded be~~ roote is found inexpressible in whole numbers yn adding ciphers & pointing them from ye Unite towards ye right hand as was before explained & soe hold on ye worke in decimalls.

As for ye Divisors they are easily found by ye 2d Table of Powers from a Binomiall roote.

If ye ~~N~~ Number bee of 6.7.8.9.10 &c dimensions The roote may be extracted after ye same manner

Of y^e Extraction of Rootes in Affected powers.

The manner of y^e extraction of rootes in pure & affected powers is verry much alike especially when y^e affected powers are decently prepared, y^t is, when theire affections are not over large & those altogather either affirmative or negative, & y^e power affirmative, affirmations & negations so mixt y^t there be noe ambiguity & all fractions & asymmetry taken away.

All y^e figures in y^e coeffents of affected power are to be pointed (after y^e manner before eplained in y^e Analisis of pure powers) according to y^e degree of theire dimensions & the worke onely differs from y^t in pure powers in y^t y^e coefficients enter into y^e divisors.

Let y^e first side be called A. y^e 2^d be called E. y^e Roote of y^e Equation L y^e coefficients B. Cq. Dc. Fqq. Gqc Hcc &c y^e Power P. Pq. Pc. Pqq &c & y^e Operation follows

The analysis of Cubick Equations.

The equation supposed $Lc + 30L = 14356197$. $Lc + CqL = Pc$

The square coefficient		3	0	
The cube affected to be	14	356	197	(243
Sollids to be substracted	8			= Ac
		6	0	= ACq
Theire summe	8	006	0	
Rests	6	350	197	for finding y^e 2^d side

The Extraction of y^e seacond side

		Coefficient	30	or superior divisor.
The rest of y^e cube to be	6	350	197	resolved
The inferior divisors	1	2		3 Aq
		6		3 A
Theire summe	1	260	30	
Sollids to be substracted	4	8		= 3 AqE
		96		= 3 AEq
		64		= Ec
		1	20	ECq
Theire summe	5	825	20	

The superior part of ye divisor		3 0	or ye Square coefficient
The remainder for finding	5 2 4	9 9 7	ye third side
	1 7 2	8	3 Aq that is 3 X 24 X 24
The inferior part of ye divisor		7 2	3 A or 3 X 24.
The sum of ye divisors	1 7 3	5 5 0	
	5 1 8	4	3 Aq E
Sollids to be taken away	6	4 8	3 A Eq
		2 7	Ec
		9 0	E Cq
Theire Suma	5 2 4	9 9 7	
Remaines	0 0 0	0 0 0	

But ye Coëfficient may be greater yn ye Power soe yt it cannot be subtracted from it wch argues yt ye Cube more propperly affects yn is affected. In this case ye coëfficient must descend towards ye unite soe many points untill it may be substracted, & soe many points as ye coëfficient is devolved soe many pricks must be blotted out towards ye left hand in ye power affected. As ye Example shews

$Lc + 95400L = 1819459.$

	9 5 4	0 0 . .	Coefficiens
	1 8 1 9	4 5 9	The Power

Since 9 is greater yn 1 make a devolution thus.

	9 5 4	0 0 .	The Coefficient
The Quote (19	1 8 1 9	4 5 9	The affected power
Sollids to be subtracted	9 5 4	0 0	A Cq
	1		Ac
Suma	9 5 5	0 0	substrahenda
Divisorū superior pars	9 5	4 0 0	Coefficiens Planum.
	8 6 4	4 5 9	Potestas reliqua
Divisorū pars inferior		3	3 Aq
		3	3 A
Divisorū suma	9 5	7 3 0	
	8 5 8	6 0 0	E Cq
Sollida ablativa	2	7	3 Aq E
	2	4 3	3 A Eq
		7 2 9	Ec
Eorū Summa	8 6 4	4 5 9	
Restat	0 0 0	0 0 0	

To place y^e unite of y^e coefficient in its right place in respect of y^e power make so many pricks above as there are under y^e power begining at y^e unit, & if y^e coefficient be one dimension lesse y^n y^e power make a prick on every figure if 2 dimensions les y^n on every other figure if 3 dimensions lesse make it one each third figure &c

If there be many coefficients in y^e Equation each must be placed according to this rule.

Some times y^e coeffient is under a negative sine as $Lc - 10L = 13584$ & y^e Analysis is as follows

Coëfficiens planum −	10	sublaterale
Cubus resolvendus +13	584.	(24
Solida ablativa { 8		Ac
{ −	20	ACq
Suma +7	80	
Restat +5	784	resolvandum
Divisorum ps superior	−10	coëfficiens planum.
Divisorum ps inferior { +1	2	+3AA
{ +	6	+3A
Suma divisorum +1	25	
Solida ablativa { +4	8	+3AAE
{ +	96	+3AEE
{ +	64	+EEE
{ −	40	+ECC
Eorum suma +5	784	

But sometimes y^e square coëfficient hath more pairs of figures y^n y^e cube to be analysed, hath threes, & y^n præfixing so many ciphers to y^e cube as figures are wanting, y^e first side will not much differ from y^e square roote of y^e coefficient. as

$Lc - 116620L = 352947.$

	−11	662 | 0	Coefficiens plane
Cubus resolvendus	00	352 | 947.	(343
Sollida Ablativa {	+27		Ac
{	−34986		ACq
Restat auferendum	−7986		
Reliquum resolvendi	+8338	947	Cubi

Divisorū ps superior	Coeff: −1	166	20.	planum.
Reliquū resolvendi cubi	+8	338	947	negative affecti
Divisorū ps inferior	+27			3AA.
	+	9		3A.
	~~279~~			~~3AA+3A+Cq~~
Suma Divisorum	+1	623	8.	3AA+3A+Cq
Sollida ablativa	+10	8		3AAE
	+1	44		3AEE
	+	64		EEE
	−4	664	8	CCE
Eorum summa	+7	639	2	
Restat Resolvend	+	699	747.	pro 3o latere
Divisorū ps superior ∾	−	116	620	CC
Divisorū ps inferior	+34	6	8	3AA
	+	1	02	3A
~~Eorum summa~~		~~231~~	~~2003~~	
Eorum Summa		231	200	=3AA+3A+CC
Sollida Ablativa	+1	040	4	3AAE
	+	9	18	3AEE
	+		27	EEE
	−	349	860	ECC
Eorum summa	+	699	747	

Sometimes though there be as many 2 figures in ye coefficient as 3 figures in ye Cube affected yet ye coefficient may be so greate as to deceive an unwary Analist As in this $\mathcal{L}c - 6400\mathcal{L} = 153000$. where ye roote of 64 is 8 wch cubed is 512 wch added to 153 makes 665 the whose roote ye number immediately greater is 9 wch ~~make~~ is ye first side = A.

But if ye coefficient had beene affirmative, yn not ye aggregate of ye facts but ye difference must be taken as in this. $\mathcal{L}c + 64\mathcal{L} = 1024$. Since ye roote of 64 is 8. wch cubed is 512. & $1024 - 512 = 512$. ye roote of wch is 8 = A. The like is observable in equations of higher powers

If ye Cube be affected wth a negative signe as $13,104\mathcal{L} - \mathcal{L}c = 155,520$. Then ye Equation is expressible of 2 roots: whereof ~~is less ye other greater yn~~ 1o ye square of one is

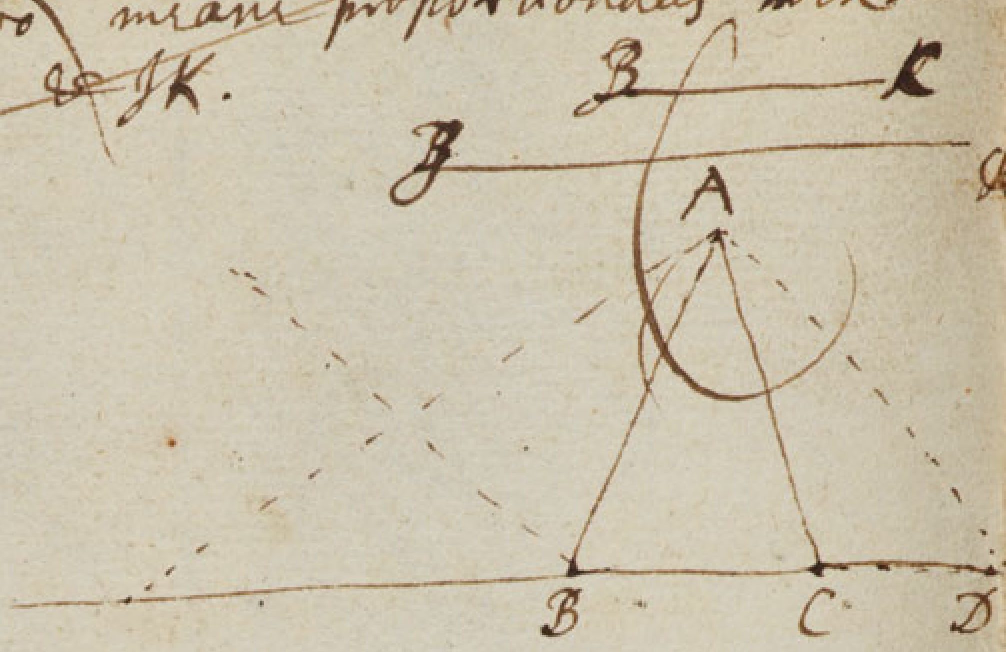
lesse & ye square of ye other is greater ~~than~~ $\frac{13104}{3}$.
& therefore one roote is lesse ye other greater then $\frac{155520}{13104}$.

& in this equation $27755\,l - lqq = 217944$ are two rootes whereof one is greater ye other lesse then $\frac{217944}{27755}$.

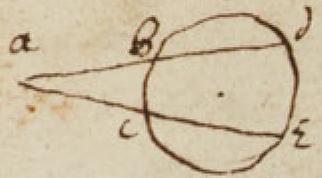
Suppose in ye former cubick equation ye lesse roote be 12. ye $\frac{155520}{12} = 12960$. or else $13104 - 12 \times 12 = 12960$. & $lq + 12\,l = 12960$. where $l = 108$ is ye greater roote.
And in ye latter equation if ye ~~lesse~~ greater roote be 27. & $\frac{217944}{27} = 8072$, c. or $-27 \times 27 \times 27 + 27755 = 8072$. $27 \times 27 = 729$. If there be 4 cubes continually proportionall whose greate extreame is $27c = 19683$. & ye aggregate of ye 3 rest is 8072 & lc ye lesse extreame, therefore $lc + 27\,lq + 729\,l = 8072$. ye roote of wch is 8 ye other roote of ye equation

Or having one roote of an equation ye Equation may be lessoned by division thus $13104\,l - lc = 155520$ Or $l^3 - 13104\,l + 155520 = 0$. & one roote is 12. therefore divide this equation by $l - 12$ & ye Quote is an equation conteining ye other roote viz: $lq + 12\,l = 12960$.

To find two meane proportionalls twixt BC & JK.



Propositiones Geometricæ. Franc: Vietæ.



prop 1

ab : ac :: ce : bd.

prop 2

& if ab : ac :: ac : bd: then ac : ab :: ab : ce

prop 3. If ab × ac = bd × ce. then bd : ac :: ac : ab :: ab : ce ∴

prop 3. To find two mean proportionalls twixt BC & gK.

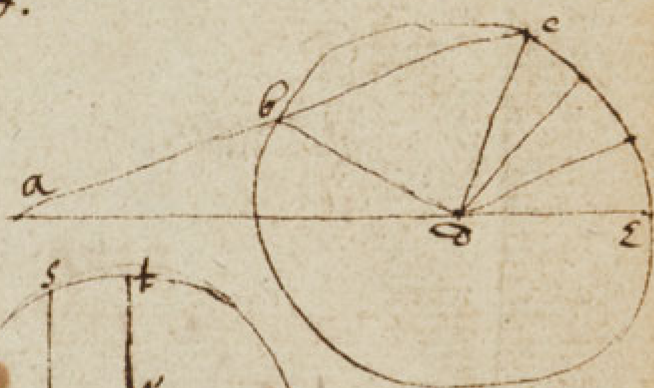
On y^e center a wth y^e Rad ai describe y^e circle ibck. inscribe bc = cd. draw da through y^e center & bg parallel to it. draw hk through soe y^t gh = ab (= ai). & ik : hb :: hb : hi :: hi : bc. ∴

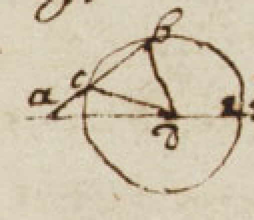
Prop: 4

If ad = db = cb. y^n y^e Angle cbe is tripple to y^e Angle abd.

Prop 5

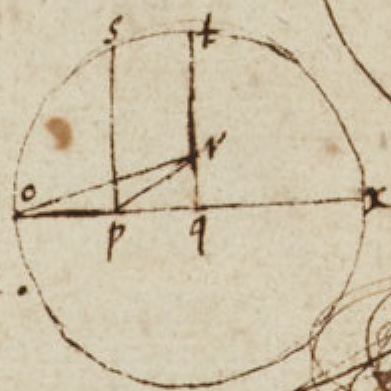
If ab = bd = Rad. 3 Ang: bad = cde.

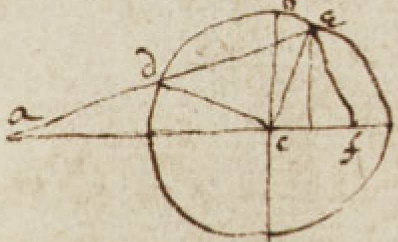




Prop 6

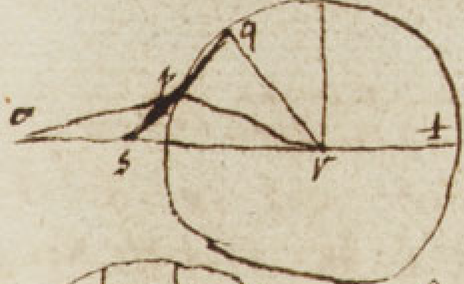
If 3 rpq = spq : recto. that is If 2 qr = pr. then 3 or × or = sp × sp + op × op + px × px.





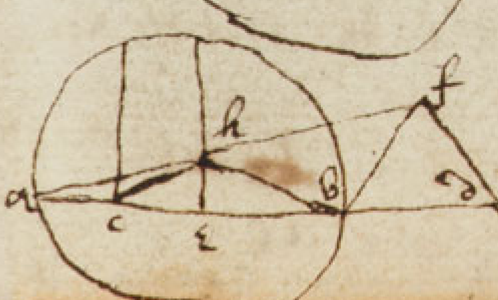
Prop 7

~~If~~ If ad = dc = ce = ef. then ecf = efc = 3 dac = 3 dca & ac3 = 3 AC × ad² + cf × ad². $\boxed{z^3 = aaz + b^3.}$ ∾ ∾ ∾

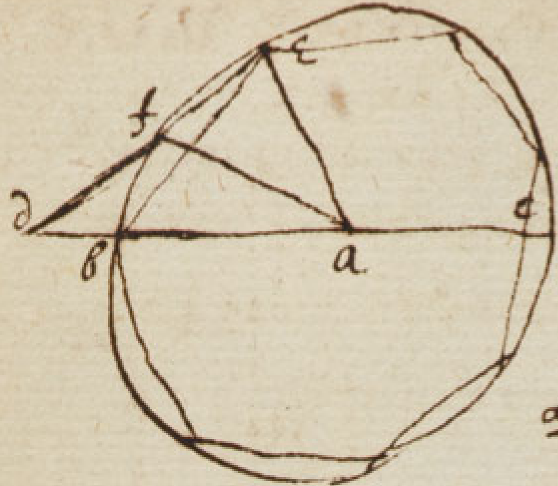


prop 8. If op = pr = qr = qs. y^n, prt = 3 qsr &c and sr3 = 3 sr × qr² − or × qr². $\boxed{z^3 = aaz - b3}$

Prop 9 If ~~op = pr =~~ ah = hb = bf = fd. & ch = 2 eh or ceh = 3 hce. then ~~bg3~~ ac³ = 3 ac × ah² − db × ah² & cb3 = 3 cb × ah² − db × ah² } $\boxed{z3 = aaz - b3.}$



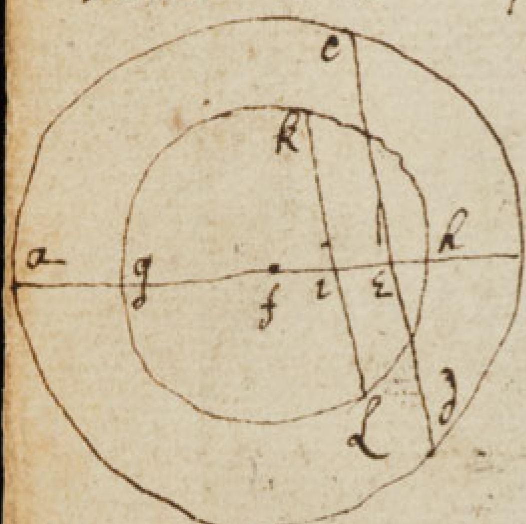
prop 10

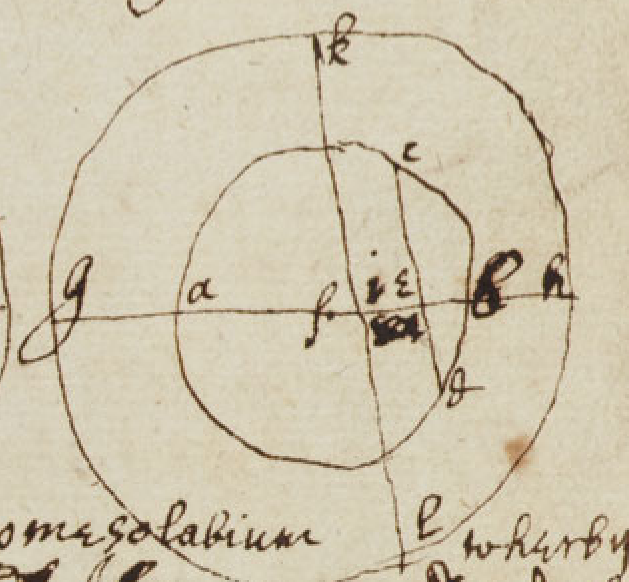


If $de = ea$ & $db : da :: ab \times ab : dc \times dc$. yn be is a side of a 7 equall sided & angled figure. or $7\,eab = 4$ right angles.

prop 10

If $ac = ef$. & aef a right angle & ab pas through ye center then $cd : de :: de : df :: df : db$. And if $cd : de :: de : df :: df : db$. then ae is perpendicular to ef. $2kd$ is ye difference of ye extreames & $2do$ is ye difference of ye meanes. wch given ye proportionall lines may be found &c.

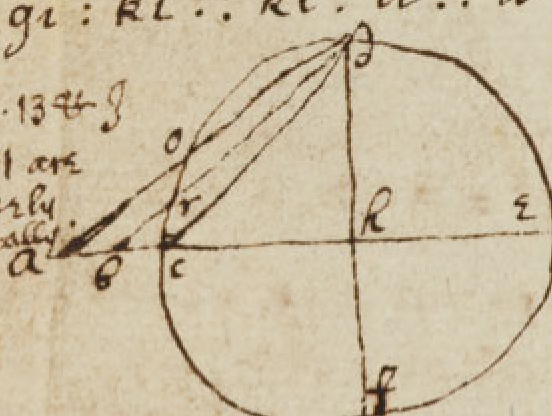




prop 11 Pseudomesolabium whereby

To find 2 meane proportionalls. ~~If~~ If $ae : ec :: ec : ed :: ed : eb$. they be inscribed in ye circle $acbd$ ye diam: being $ae + eb$. If twixt gi & ik two meane proportionalls are sought on ye same center f wth ye Rad: $\frac{gi + ik}{2}$ describe $gkhl$ & inscribe a line kl parallell to cd cutting ab in ye point i & $gi : ki :: ki : il :: il : ik$. Examine it.

prop 12 · 13 &c I think they are trew only mechanically.

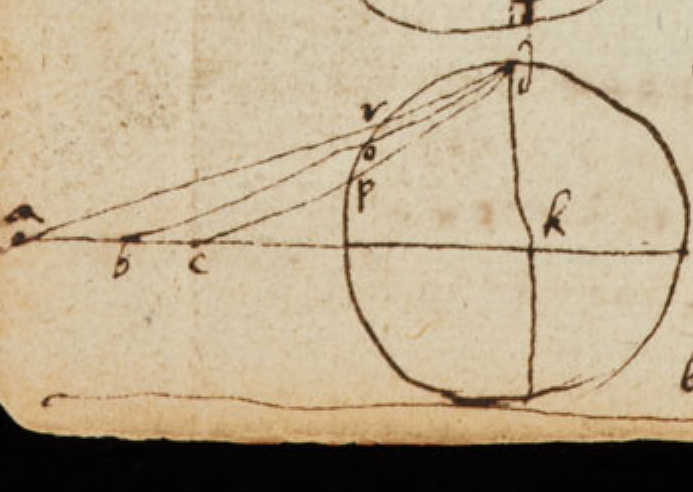


prop 12

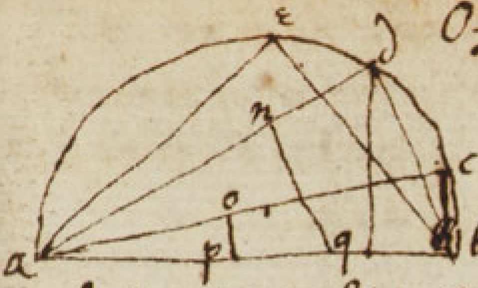
If $do = dk$ & ac bisected in b & bd bee drawne rd is ye side of a pentagon wch may be inscribed in $defcro$

prop 13.

If rd be ye side of a {octogon / decagon} & pd ye side of an {hexagon / octogon} ye arch rp divided ~~equally~~ in o, od will be the side of an {heptagon / enneagon} to be inscribed in ye circle ord & ye arch RP is rightly divided by Bisecting ye Line ac.



Examine it



Of Angular sections.

Prop 14. If $\angle$ ead = cab. Then

$ab : ab^2 :: cb : eb \times ad - ae \times db :: ac : ae \times ad + eb \times db$

Or, $ap : aq \times ab :: op : eb \times an - ae \times nq :: ao : ae \times an + eb \times nq$.

But ye angles anq, aop are right ones & ean = oap = eab - dab

Prop 15

If ye angle cab + dab = eab. or naq + oaq = eab. & anq, aop are right angles

then $Ab : ab^2 :: Eb : ad \times bc + ac \times db :: ea : ad \times ac - db \times cb$.

Or the triang: unequall. $ab : ap \times aq :: eb : an \times op + ao \times nq :: ea : an \times ao - nq \times op$

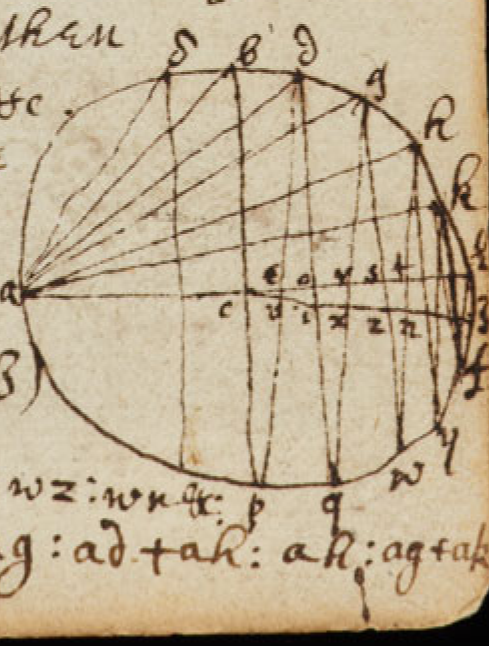
Prop 16.

In 2 rectang: triang: acb & aed, if ye first have an acute angle cab submultiple to ye acute angle eab of ye 2d triang aeb ye sides of ye seacond have this proportion. suppose ye Hypoten of ye first tri: bee Z. ye base B. ye Cathetus C.

If ye acute angle of ye seacond Triangle bee to ye acute angle of ye first triangle in a proportion

	Hypoten:	Base.	Perpendicular
Duple,	Z^2.	B^2 $-C^2$.	$2BC$.
Triple,	Z^3.	B^3 $-3BCC$	$3BBC$. $-C^3$.
Quadruple,	Z^4.	B^4. $-6B^2C^2$. $+C^4$.	$4B^3C$. $-4BC^3$.
Quintuple,	Z^5.	B^5. $-10B^3C^2$. $+5BC^4$.	$5B^4C$. $-10B^2C^3$. $+C^5$.

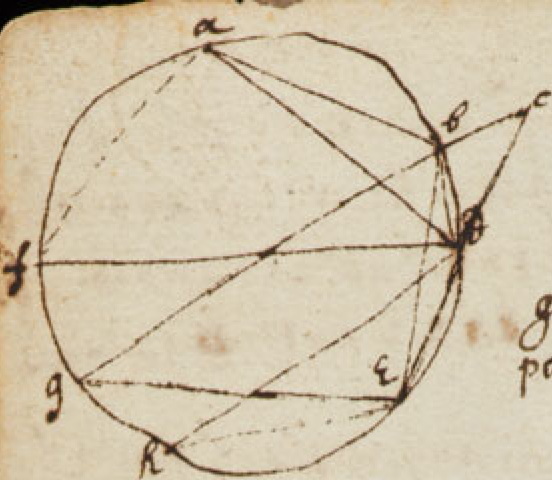
Prop 17. If $\angle$ ab = bc = ce = eg &c: & ac = cd. & ae = ef &c then $ab : ac :: ac : ad :: ae : af$ &c. & ed = ab. & ac = gf. &c. nam Δcde & cba, efg & eac &c = & sim.



Prop 18.

If bd = dg = gh = hk = pq = pw &c

Then $al : ak :: pe : pc :: ed : do :: pd : pc + do ::$ $rg : gs :: rq : qo :: qg : qo + gs ::$ &c & if $\left(\frac{lf}{2} = lz\right)$ from z to ye center bee drawne cz then $al : ak :: di : dv :: zq : qx :: dq : dv + qx :: gz : gx :: wz : wn$ &c. & Ergo $ac : ak :: ab :: ab + ad : ad : ab + ag : ag : ad + ak : ak : ag + ak$ &c.



Prop 19 If $fa = ab = b\varepsilon = \varepsilon k$ &c. $af + ab + b\varepsilon + \varepsilon k$ are greater yn ye semiperiphery: ~~then, Rad: ge (= εε)(= ad) :: bd (= bε) : εd (= ad − dε) :: dε : ddk − dε~~ & dk is ye greatest, db ye least line drawn from d to those points a, b, ε, k. yn rad: dk :: db : da − dε.

prop 20 Out of ye 18th & 19th Prop:

To divide an angle into any number of pts in ye figure of ye 18th prop: $al = \text{diam} = 2z$. ak is ye greatest of ye inscribed lines $= B$: now $2z : B :: B : ak + 2z$. therfore $bb = ak \text{ in } z + 2z^2$. & $\frac{bb - 2zz}{z} = ak$. And $z : B :: \frac{b^2 - 2z^2}{z} : b + ag$. therfore

$\frac{b^3 - 2zzb}{zz} - b = ag$ or $\frac{b^3 - 3zzb}{zz} = ag$ Likewise

$\frac{B^4 - 4zzbb + 2z^4}{z^3} = ad$. & $\frac{B^5 - 5zzB^3 + 5z^4B}{z^4} = ab$

$\frac{B^6 - 6zzB^4 + 9z^4BB - 2z^6}{zzzzz} = ad$

$\frac{B^7 - 7zzb^5 + 14z^4b^3 - 7z^6b}{z^6}$ = to a seaventh line

$\frac{B^8 - 8zzb^6 + 20z^4b^4 - 16z^6bb + 2z^8}{z^7}$ = to an eighth line

$\frac{B^9 - 9zzb^7 + 27z^4b^5 - 30z^6b^3 + 9z^8b}{z^8}$ = a ninth line

$\frac{B^{10} - 10zzb^8 + 35z^4b^6 - 50z^6b^4 + 25z^8bb - 2z^{10}}{z^9}$ = tenth &

Prop 21 out of ye 17th Theor: in ye figure whereof if ab ye least inscribed line $= z$. & ae ye next line bee B. then $z : B :: B : z + a\varepsilon$. & $\frac{bb - zz}{z} = a\varepsilon$. & $\frac{B^3 - 2z^2B}{zz} = ag$. & $\frac{B^4 - 3z^2bb + z^4}{z^3}$ = to a fift line.

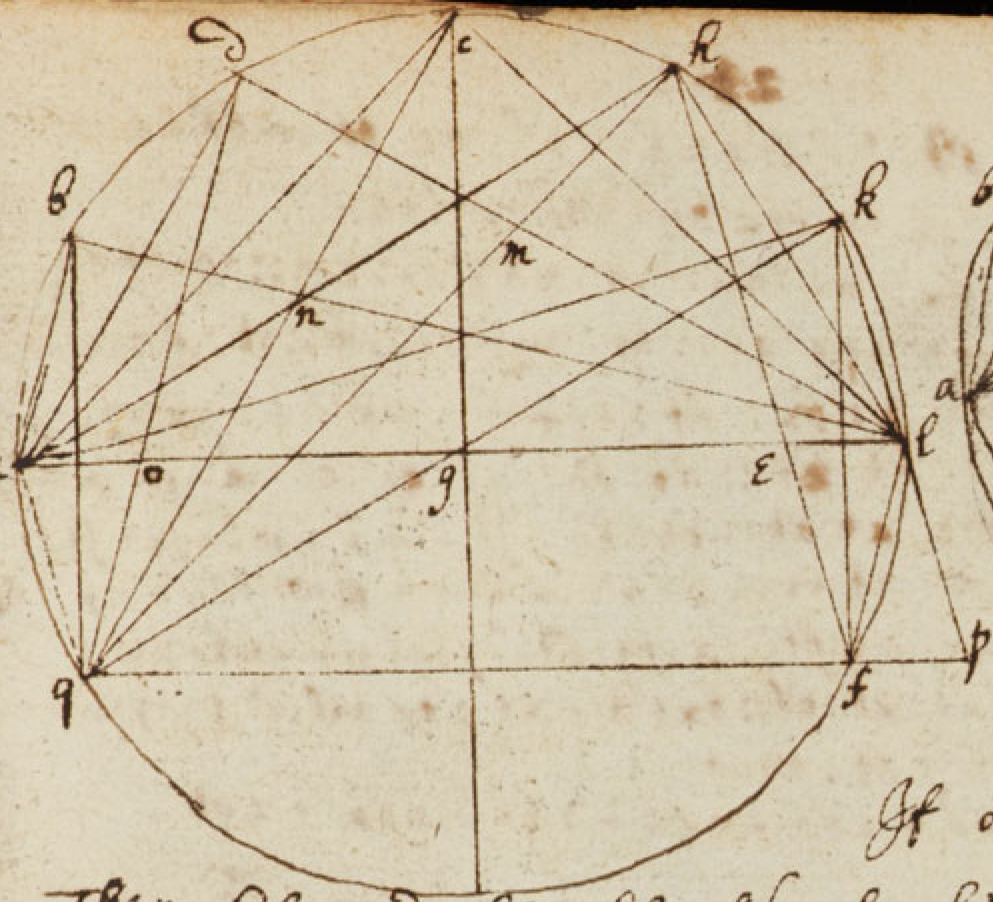
$\frac{B^5 - 4zzb^3 + 3z^4b}{z^4}$ = a sixt. & $B^6 - 5zzb^4 + 6z^4bb - z^6$ = seaventh

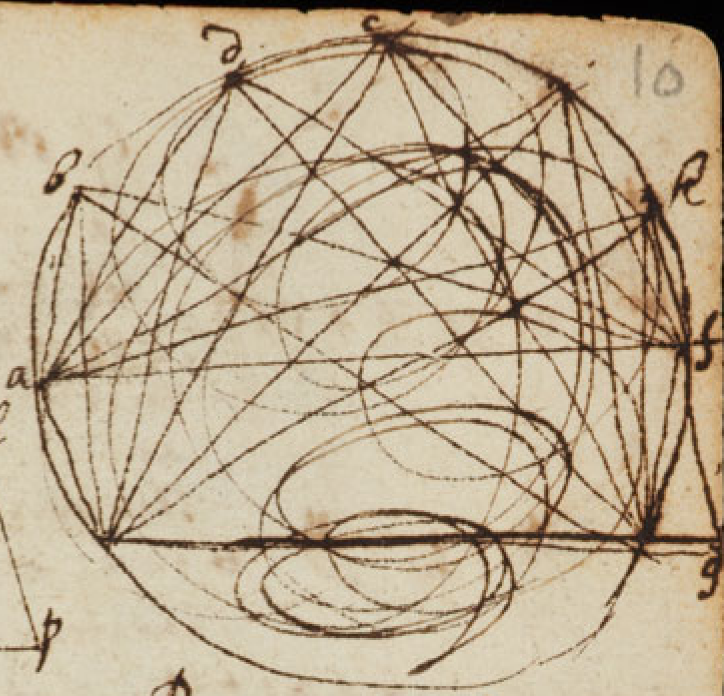
$\frac{B^7 - 6zzb^5 + 10z^4b^3 - 4z^6b}{z^6}$ = to an eight line

$\frac{B^8 - 7zzb^6 + 15z^4b^4 - 10z^6bb + z^8}{z^7}$ = to a nineth line

$\frac{B^9 - 8zzb^7 + 21z^4b^5 - 20z^6b^3 + 5z^8b}{z^8}$ = to a tenth line

$\frac{B^{10} - 9zzb^8 + 28z^4b^6 - 35z^6b^4 + 15z^8bb - z^{10}}{z^9}$ = Eleventh.





Prop 22 .

If $aq = ab = bd = dc = ch = hk = kl = lf$.

Then Gk rad : $kl :: kl : \varepsilon l\ (= al - ah) :: hl : hm\ (= qh - qd = = ak - ac) :: dl : do\ (= qd - qa = ac - ab) :: lc : cn\ (= qc - qb = ah - ad$ &c) for that ~~[illegible]~~ ye Periph: divided into any number of pts. $gl : lk :: lk : al - ah :: lh : ak - ac :: lc : ah - ad :: dl : ac - ab$ &c. & $gl : lk :: ah : lc - lk :: ac : ld - lh :: ad : lb - lc :: ab : al - ah$ &c.

Hence Prop 23. In ye former schreame

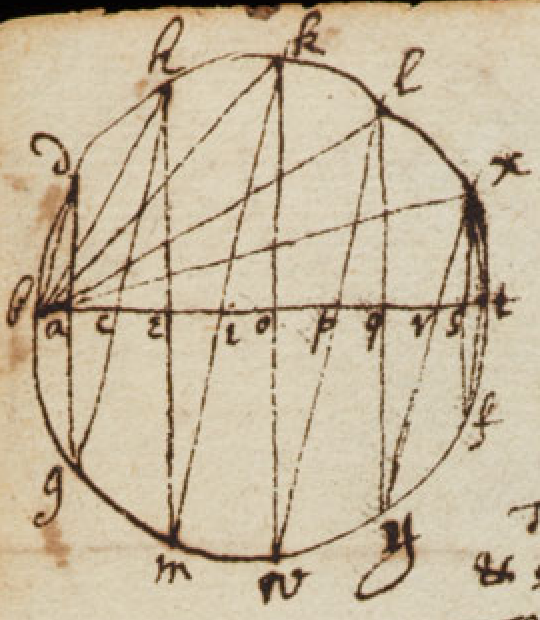
If $al = 2x =$ hypotenusa. $kl = b$. $x : b :: b : 2x - ah$ & $\frac{-bb + 2xx}{x} = ah$ / $x : b :: \frac{-bb + 2xx}{x} : \frac{2bxx - b^3}{xx}\ (= lc - b)$ ~~[illegible]~~

therefore $\frac{3bxx - b^3}{xx} = lc$. & $\frac{2x^4 - 4bbxx + b^4}{x^3} = ad$ ye base of ye 4th triang: & $\frac{5bx^4 - 5b^3xx + b^5}{x^4} =$ ye perpendicular (bd) of ye 5t tri: & $\frac{2x^6 - 9x^4bb + 6xxb^4 - b^6}{x^5} =$ base of ye 6t triang.

$\frac{7x^6b - 14x^4b^3 + 7x^2b^5 - b^7}{x^6} =$ perpendic of ye 7th tri

$\frac{2x^8 - 16x^6bb + 20x^4b^4 - 8xxb^6 + b^8}{x^7} =$ base of ye 8th tri:

$\frac{9x^8b - 30x^6b^3 + 27x^4b^5 - 9xxb^7 + b^9}{x^8} =$ perp: of ye 9th tri:



Prop ~~24~~ :

If ~~bd~~ = dh = hk = kl = bg &c: then bh = gd & bk = gh & bl = hm &c: & then xt : bx :: ac : ag :: ce : eh :: ei : em :: io : ok :: op : ow :: pq : ql :: qr : qy :: rs : sx :: st : sf :: ba : ad

therefore xt : bx :: bt : dg + hm + kw + ly + xf.

againe xt : bt :: ab : bd :: ac : cg :: ce : eh :: ei : im &c

Therefore xt : bt :: bt : bd + gh + mk + wl + yx + ft.

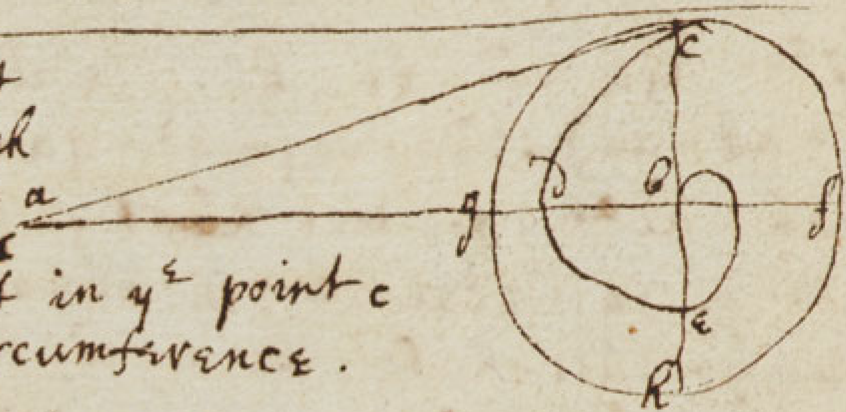
& since, as xt : bt :: bx : dg + hm + kw + ly + xf

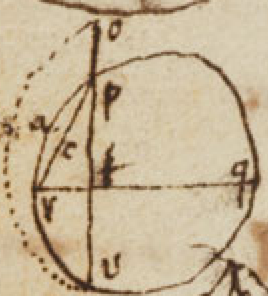
Therefore xt : bt :: bx + bt : bd + dg + gh + hm + mk + kw + wl + ly + yx + xf + ft. And xt : bt :: bx + bt + xt : to all ye perpendicular & transverse line + bt. that is

xt : bt :: xt + bt + bx : 2bd + 2bh + 2bk + 2bl + 2bx + 2bt.

Prop 24

If in ye circle efgh be inscribed ye helix bedc & ac touch it in ye point c then ab = to ye circumference.





Prop 25

If apcr be les yn halfe ye circle, & vt = tp. & vo = to vrap: then $\frac{rq \times po}{2}$ = 4 times ye section (rapc

Prop 26

If ab = bd = ad & ~~bk~~ perpendicular to ad fro ye angle b. ce = ed. yn aed = adi = 3 dae. & ed is ye side of a heptagon

Prop 27.

If a line be cut by extreame & meane proportion ye lesse segment almost is to ye whole line as ye diameter is to 5 times ye Periphery divided by 6.

Prop 28

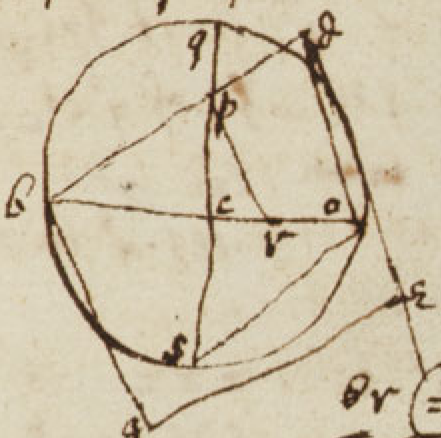
Si secetur linea per extremam & mediam proportionam erit proxime, ut tota linea plus minori segmento ad bis totam lineam, ita quæ potest quadrato sesquialterum semidiametri, ad latus quadrati circulo equalis.

linea secta sit 100,000. minus segmentū 38,197. Semidiamet 100,000, quæ potest quadrato sesquialterū semidiametri paulo major est quàm 122,474. Radix Peripheriæ, 177,245.

Prop 28.

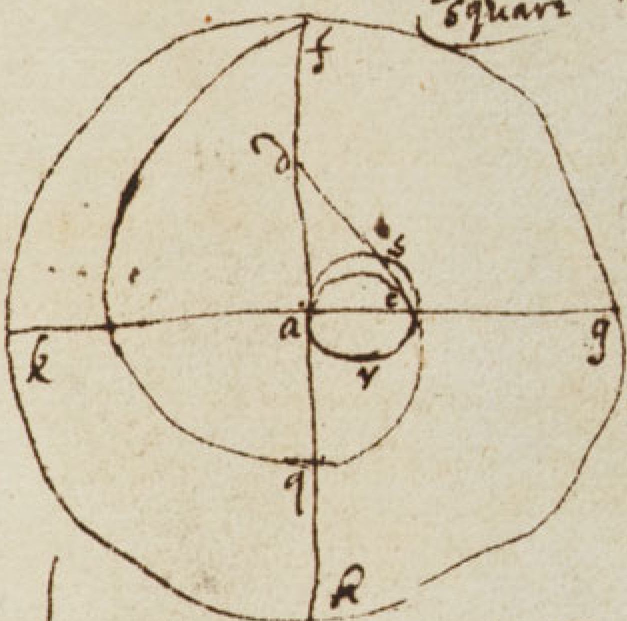
If εr = rh = or. & ao = fc = to y^e side of a decagon; & fn parallell to cd y^n εn shall be almost equall to y^e fourth p^t of a circle for af is divided in extreames & meanes propor in y^e point c. & εc : εf :: ~~...~~ εf : $\frac{5}{12}$ Perim ~~...~~ fa :: hr : $\frac{5}{24}$ Perim :: $\frac{6}{10}$ εf (= ~~...~~ ε) :: $\frac{1}{4}$ Perim; by y^e & εc : εf :: εd : εn (= $\frac{1}{4}$ Perim).

27^th prop:



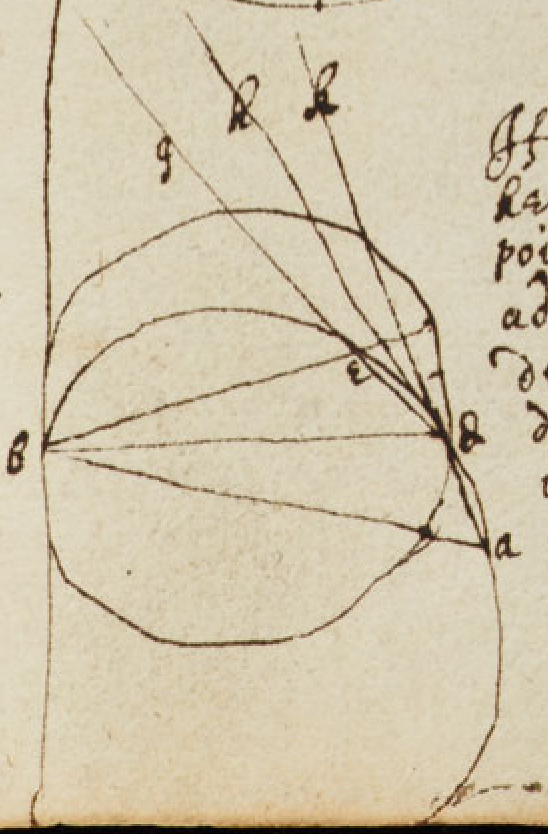
Prop. 29.

If os = 2cp. & co is divided by extreame & meane proportion in r. & od parallell to rp then ∂b is y^e side of a square = to the area of y^e circle. for by y^e 28th prop: os br (= to line + less segm) : bo (= twice y^e line) :: bp (= $\sqrt{\frac{3}{2}}$ of y^e ~~Diam~~ square of y^e semidiameter) : bd (= to y^e roote of a ~~figure~~ square equall to y^e area of a circle.



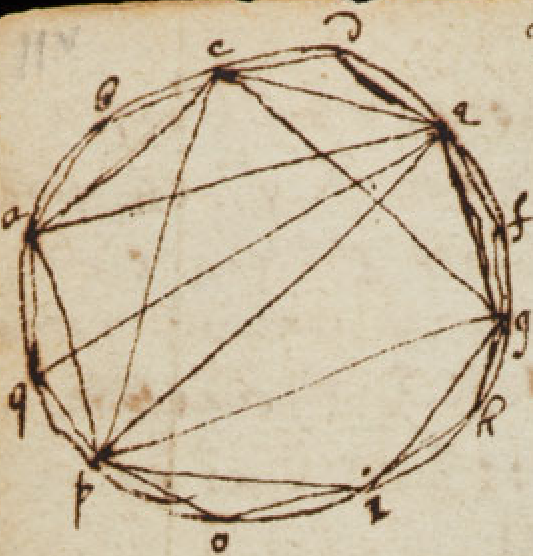
Prop 30

If y^e line dc touch y^e helix in y^e line ag. & y^e line hf toucheth y^e beginning of it in y^e center a & 4ac = af then 2ad shall bee equall to perim: asr. & ac being y^e Diam: y^e ~~triangle~~ area of y^e triang acd = to y^e area of y^e circle asr



Prop 31.

If bεd be a square of one revolution of an helix & y^e angle dbε = dba & through y^e points a, d, in y^e helix be drawne y^e line adk & through y^e points εd in y^e Helix be drawne εdg. & y^e angle kdg bisected by dh; then dh shall almost touch y^e helix in d. ~~&~~ it shall be soe much y^e nigher a touch line by how much y^e angles εbd dba are lesser.



Prop 32

If many Polygons bee inscribed in a circle ye number of theire sides increaseing in a double proportion & theire apotomies, or ye base of a tri: whose cathetus is a leg of ye Polygon & hypotenusa is ye Diam (as ye apotome of ye Polygō egp is ce. of paeegi is ae &c) if ye Apotome of ye sides of ye first Polygō be called b. of ye 2d = c. of ye 3d = d. of ye 4th = f. of ye 5t = g of ye sixt = h. & ye diameter be z ~~then~~ And ye first Polygon be = p. ye 2d = q. ye 3d = r = abcdefghiopq. ye fourth = s. ye 5t = t ye sixt = v. ye 7th = w &c then $p:q::b:z$. & $p:r::bc:zz$. & $p:s::bcd:zzz$ & $p:t::bcdf:z^4$. & $p:v::bcdfg:z^5$. & $p:w::bcdfgh:z^6$ &c

To know how many ~~Elections may bee made of~~ things, wherof some of ym are equall, divers ways may bee ordered. as of, a.b.b.c.c.c.d.d. doe thus $\overset{a}{\frac{1}{1}}\times\overset{b\ b}{\frac{2\times3}{1\times2}}\times\overset{c\ c\ c}{\frac{4\times5\times6}{1\times2\times3}}\times\overset{d\ d}{\frac{7\times8}{1\times2}}=112$ the number of changes, in order.

To know how many Elections may bee made doe thus a bb ccc dd eeee $2\times3\times4\times3\times5=360=\overset{a}{2}\times\overset{b\ b}{\frac{2\times3}{2}}\times\overset{c\ c\ c}{\frac{2\times3\times4}{2\times3}}\times\overset{d\ d}{\frac{2\times3}{2}}\times\overset{e\ e\ e\ e}{\frac{2\times3\times4\times5}{2\times3\times4}}$ therefore there are $359=360-1$ Elections in $abb c^3 dd e^4$.

Propositiones Geometricæ Ex Schootenij Sectionibus miscellaneis. 12

Sectio 1ma

To know how many changes 6 Bells, abcdef. or how divers conjuctions ye 7 Planets can make ♄ ♃ ♂ ☉ ♀ ☿ ☽. or how many divisors abcde hath, or how many divers compositions ye 24 letters can make &c the examples following show.

1. a 1
2. b. ab 3
4. c. cb. cab. ac 7
8. d. da. ~~db~~. dab. dc. dac. dcb. dcab. 15
16. e. ea. eb. eab. ec. eac. ecb. ecab. ed. eda. edb. edab. edc. edac. edcb. edcab. 31
32. f. fa. fb. fab. fc. fcb. fac. fcab. fd. fda. fdb. fdab &c 63.
64. g. ga. gb. gab. gc. gcb. gac. gcab. gd. gda. gdb &c 127

wch shows yt in 7 letters 127 elections may be made yt 7 Planets may be conjoyned 120 divers ways. yt abcdefg. hath 128 divisors for an unite is one of them. & $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$, are ye number of changes in six bells.

Sec 2.

To know how many things & of wt sort they are wch may be chosen 15 ways. $15+1=16$. $\frac{16}{2}=8$. $\frac{8}{2}=4$. $\frac{4}{2}=2$. $\frac{2}{2}=1$. & ~~4~~ $2-1.2-1.2-1.2-1.=1.1.1.1.=4$. that 4 things all unequall may be varyed 15 ways. also. $\frac{16}{4}=4$. $\frac{4}{2}=2$. $\frac{2}{2}=1$ $4-1.2-1.2-1=5$ & 5 things whereof 3 are equall viz: a.a.a.b.c. & $\frac{16}{4}=4$. $\frac{4}{4}=1$. $4-1+4-1=3+3=6$. & 6 things whereof 3 & 3 are eaquall as aaabbb. may be varyed 15 ways. & $\frac{16}{8}=2$ $\frac{2}{2}=1$. $8-1.2-1=8=7+1$. & 8 things whereof 7 are = may be varyed 15 ways. as aaaaaaab. & $\frac{16}{16}=1$. $16-1=15$. wherefore 15 alike things &c as a15. 2 Wt things vary 23 ways. $23+1=24$ ~~&~~ 24 admitts a 7fold divison therefore ye multitude of things sought may be 7 fold but since 43 is a primary number (viz wch cannot bee divided) $42+1=43$. $\frac{43}{43}=1$ $43-1=42$. therefore onely 42 like things can be varyed 42 ways as a^{42}.

Sec 3

Every quantity hath one divisor more y^t it hath aliquote pts (y^t is pts ~~greater y^n an unite~~ of whole numbers). How to find a quantity haveing a given multitude of divisors or aliquote pts: suppose its aliq: pts must be 15. $15+1=16$ & soe by y^e former section $abcd$. a^3bc. a^3b^3. a^7b. a^{15}. may be varyed 15 ways therefore they shall have 15 aliquote pts & 16 divisors. but since onely 42 like things (as a^{42}) can be varyed 42 ways therefore onely a^{42} hath 42 aliquote pts & 43 divisors. &c

Sec 4

To find y^e least numbers haveing a given multitude of divisors & aliquote pts instead of soe many letters in y^e former sec: put soe many ~~pr~~ least primary numbers ~~& take~~ y^e least result from y^m. as from y^e former example: $abcd$. a^3bc. a^3b^3. a^7b. a^{15}. that is 2.3.5.7. or 2.2.2.3.5. &c. now. $2\times3\times5\times7.=210$. & $2\times2\times2\times3\times5=120$. &c therefore $2\times3\times5\times7.=210$ is y^e least number haveing 16 divisors.

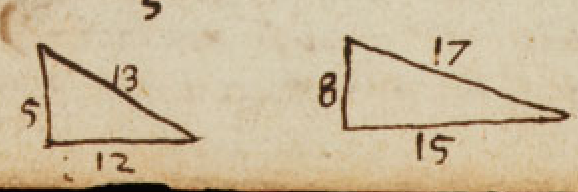
Sec: 5 conteines a table of Primary numbers.

Sec 6

To find progressions constituteing rectangular triangles w^{th} sides rationall y^e examples following shew. take two numbers as 1. 2. y^m $1\times2=2$ since y^e product is even double it viz: $2\times2=4$. & 4 is y^e numerator y^n $1+2=3$ & since 3 is od multiply it by y^e difference of y^e termes: $1\times3=3$ & 3 is y^e denominator. & y^e first terme $\frac{4}{3}$. y^n since (1) y^e difference of y^e termes is od multiply it by 4. $4\times1=4$ & $4\times$ per 2 majors terminum. $4\times2=8$ $8+4$ (the former numerator) $=12=$ numerator 2^d. y^n 3 (y^e former denom) added to 2 (y^e double square of y^e diff: of y^e termes because y^e square (1) is odd) $=5$ y^e 2^d denominator. & ad another example take 1. 3. y^n $1\times3=3=1^{st}$ numerator. y^n $1+3=4$ & since 4 is even $\frac{4\times2}{2}$ (diff: of y^e termes) $=4$ & y^e ~~halfe ... 1^{st} denom~~ first denom is 4. y^e first terme $\frac{3}{4}$. y^n becaus y^e diff of y^e termes is even 2 $2\times2=4$ & $4\times3=12$ & $12+3=15$. y^n $2\times2=4$. $4+4=8$. & $\frac{15}{8}$ y^e 2^d terme & now ~~termes~~ may be had by arithmeticall proportion. thus.

$\frac{4}{3}$. $\frac{12}{5}$ or $1\frac{1}{3}$. $2\frac{2}{5}$. $3\frac{3}{7}$. $4\frac{4}{9}$. $5\frac{5}{11}$. $6\frac{6}{13}$ $7\frac{7}{15}$ $8\frac{8}{17}$ $9\frac{9}{19}$ $10\frac{10}{21}$ &c

& $\frac{3}{4}$. $\frac{15}{8}$ or $\frac{3}{4}$ $1\frac{7}{8}$. $2\frac{11}{12}$. $3\frac{15}{16}$. $4\frac{19}{20}$. $5\frac{23}{24}$. $6\frac{27}{28}$. $7\frac{31}{32}$. $8\frac{35}{36}$ &c thus may other progressions be obteined. For y^e Vse take y^e numerator for one leg & y^e denom for another & y^e Hypoten: will be rationall as in $2\frac{2}{5}$ ~~$5\times2=10$ $10+2=12$~~ or $\frac{12}{5}$ $\sqrt{144+25}=\sqrt{169}=13$. & in this $1\frac{7}{8}$ or $\frac{15}{8}$ $\sqrt{225+64}=17$.



If y^e supposed numbers be 2.5. y^n $2\times5=10$. $10+10=20$. & $2+5=7$. $3\times7=21$. so y^t $\frac{20}{21}$

y^n ~~double~~ $4\times3=12$. $12\times5=60$. $60+20=80$. & $3\times3=9$. 9 doubled $=18$. $18+21=39$.

& y^e 2 first termes $\frac{20}{21}$. $\frac{80}{39}$ or $2\frac{2}{39}$. Againe, if y^e numbers be 3.4

$3\times4=12$. $12\times2=24$. & $3+4=7$. $1\times7=7$ therefore $\frac{24}{7}$. y^n $4\times1=4$. $4\times4=16$

$16+24=40$. & $1\times1=1$. $2\times1=2$. $7+2=9$ therefore $\frac{40}{9}$ is y^e 2^d & y^e progress

may be continued, as $\frac{20}{21}$. $2\frac{2}{39}$. $3\frac{5}{57}$. $4\frac{8}{75}$. $5\frac{11}{93}$. & $3\frac{3}{7}$. $4\frac{4}{9}$. $5\frac{5}{11}$. $6\frac{6}{13}$ &c.

Sec 7

To find a w^{ch} divided by 7 leaves 2. by 11 leaves 1. by 13 leaves 9. the least common divisor of 7.11.13 is $7\times11\times13=1001$. divide 1001 twice by each & consider y^e remainder of y^e second division thus.

~~$\frac{210}{2}$ | 105 | 52. because 1 is left 105 is y^e multiplyer.~~

~~$\frac{210}{3}$ | $\frac{70}{3}$ | 23. since 1 is left 70 is y^e multiplyer.~~

~~$\frac{210}{5}$ | $\frac{42}{5}$ | 8 since more y^n 1 is left (viz 2) multiply 2 till it divided by 5 leaves (1) $\frac{2\times3}{5}=1\frac{1}{5}$. therefore $42\times3=126$ is y^e multiplyer.~~

~~$\frac{210}{7}$ | 30 ($4\frac{2}{7}$. since more y^n 1 is left~~

$\frac{1001}{7}$ ($\frac{143}{7}$ ($20\frac{3}{7}$.

1 Since more y^n 1 is left (viz 3) multiply 3 till it divided by 7 leaves 1. $\frac{5\times3}{7}=2\frac{1}{7}$ therfore $5\times143=715$ y^e multiplyer.

$\frac{1001}{11}$ ($\frac{91}{11}$ ($8\frac{3}{11}$.

2 Since more y^n 1 is left (viz:3) $\frac{3\times4}{11}=1\frac{1}{11}$ therefore $4\times91=364$ y^e multipl:

$\frac{1001}{13}$ ($\frac{77}{13}$ ($5\frac{12}{13}$.

3 If but 1 had beene left 77 had beene divisor but now $\frac{12\times12}{11}=13\frac{1}{11}$. therfore $12\times77=924$ is multiplyer.

now the number sough is thus found.

Divisor.	Reliq: Multip.
7.	$2\times715=1430$.
11.	$1\times364=364$.
13.	$9\times924=8316$.
The Sume	10110.

Lastly divide $\frac{10110}{1001}$ ($10\frac{100}{1001}$ by y^e least com. divis: wherefore 100 y^e number left is y^e number sought.

Sec: 8.

Touching y^e Method of weights suppose a man have weigs of ~~[illegible]~~ 1.2.4.8.16.32 pounds &c by y^m all intermediate pounds may be thus weighed

1.	2	3.	4	5	6	7	8	9	10	11	12	13	14	&c
1.	2	1+2	4.	1+4.	2+4.	1+2+4.	8.	8+1	8+2.	8+1+2.	8+4.	8+4+1.	8+4+2	

or if his weights be 1.3.9.27.81. all weights may be supplyed thus.

1.	2.	3.	4.	5	6	7	8	9	10	11	12	13	14	&c
1	3−1.	3	3+1.	9−1−3.	9−3.	9+1−3.	9−1.	9.	9+1.	9+3−1.	9+3.	9+3+1.	27−9−3−1	

Note y^t weight marked w^{th} − signifie y^e wigh to be put in y^e opposite ~~scale~~ ballance.

Sec. 9.

To find numeri amicabiles that is 2 numbers whose aliquote pts are mutually equall to theire wholes. take this Des-Cartes his rule If (2) or any other number produced out of 2 as 2×2. 2×2×2 &c (viz 2.4.8.16.32 &c) bee such a number yt 1 taken out of it triple there rests a primary number, & yt if 1 taken from it sextuple there rests a primary number, & if 1 taken from its square octodecuple a primary number rests: ye multiply this last prime number by ye assumed number doubled & ye product is one amicable number & ye aliquote pts of it make ye other Example. if 2 be taken. 2×3−1=5 numero primario primo. 2×6−1=11 numero primario scdo. 2×2×18−1=71 numero primario tertio. 4×71=284, one amicable number, & ye 2 former prime numbers × one another & ye product ×4 ye double of ye assumed number viz 5×11=55. 55×4=220. Thus from 8. & 64 &c may be deduced amicable numbers.

Sec 10

To find triangles whose sides, segments of theire bases, & Perpendiculars are expressible by rationall numbers

1st if ye perpendic: is without ye tri: let $ac=z$. $bd=x$ $cd=y$. $ad=z+y$. $ad=y+b$. $xx+yy=yy+2by+bb$. $y=\frac{xx-bb}{2b}$. & $cd=z+y+a$. $xx+zz+2yz+yy=zz+yy+aa+2zy+2za+2ay$. $2ay=xx-aa-2za=\frac{axx-abb}{b}$. $bxx-baa-2zab=axx-abb$. $\frac{bxx-baa-axx+abb}{2ab}=z$.

puting any numbers for a, b, & x; y & z may be found. then $ad=z+y=\frac{xx+bb}{2b}$. $cd=z+y+a=\frac{xx+aa}{2a}$. wch reduced to ye common denominator 2ab; & yt cast away. $cd=bxx+baa$. $ad=axx+abb$. $de=2abx$. $ae=axx-abb$. $ce=bxx-baa$. $ac=bxx-axx+abb-aab$.

In like manner if ye perpendicular fall wthin side. $ab=bxx+baa$. $bd=2abx$. $ad=bxx-baa$. $dc=axx-abb$. $bc=axx+abb$. $ac=bxx+axx-abb-baa$.

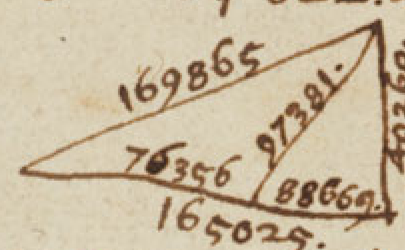
Also by ye conjunction & disjunction of 2 triangles it may be found yt $ab=bbx+aax$ $ad=bbx-aax$. $ac=bbx-aax-axx+abb$. $bc=axx+abb$. $db=2abx$. $dc=axx-abb$. For if $bd=x$ $dc=\frac{xx-bb}{2b}$. $bc=\frac{xx+bb}{2b}$. that is $bd=2bx$. $dc=xx-bb$. $bc=xx+bb$. Likewise $bd=2ab$. $ad=bb-aa$. $ab=bb+aa$. 2abx ye least quantity divisible by 2bx & 2ab, being divided by ym, leaves a & x wch must multiply ye bases & hypotenusas. If ye Perpendic: fall wthout ye legs may be thus exprest $cd=acc+ayy$. $da=yyc+aac$

$ca = acc - ayy + cyy - aac$. $ae = yyc - aac$. $ce = acc - ayy$. $ad = {}^2acy$.

Sec 11

To make y^t two such tri: be of y^e same base & altitude. Suppose an equation twixt y^e bases & perpendiculars of y^e 2 last tri: as $2abx = 2acy$. $x = \frac{cy}{b}$. $xx = \frac{ccyy}{bb}$. $bbx - aax - axx + abb = acc - ayy + yyc - aac$ or $\frac{bbcy - aacy}{b} - \frac{accyy}{bb} + abb = acc - ayy + yyc - aac$ & $yy = \frac{+b^3cy - aabc + aabbc + ab^4 - abbcc}{bbc + acc - abb}$.

Suppose $aabbc + ab^4 = abbcc$. or $a = c - \frac{bb}{c}$. let $c = 3$ greater y^n $b = 2$. $a = \frac{5}{3}$. $y = \frac{22}{61}$. $x = \frac{33}{61}$ & consequently



Sec 14 differs not from Cap 19: prob 18 Oughtred.

Sec: 15 Of Polygons or multangular numbers

The summe of all y^e tearmes in an arithmet: progres: increasing from an unite by 1 composeth ~~tria~~ triangles. By 2, composes □s. By 3, composes pentangles. By 4, hexang: &c ~~&c~~ as 1.2.3.4.5.6. compos y^e triangles 1. 3. 6. 10. 15. &c likewise 1.3.5.7.9. compose 1. 4. 9. 16. 25. &c So 1.4.7.10.13 compose y^e quintangles 1.5.12.22.35.51.70. &c. If $a = 1 =$ y^e first terme to y^e polygon y^e excess of y^e progression $= x$. y^e summe of y^e termes $= z$. y^e multitude of y^e termes $= t =$ to y^e side of y^e Polygon. ~~z given to find t~~ Suppos t given to find z. $z = \frac{1tt + 1t}{2}$ or $z = \frac{tt + t}{2}$ in trigons. $z = tt$ in 4gons. $z = \frac{3tt - t}{2}$ in 5gons. $z = \frac{2tt - t}{\cancel{}}$ in 6gons $z = \frac{5tt - 3t}{2}$ in 7gons. $z = 3tt - 2t$ in 8gons. $z = \frac{7tt - 5t}{2}$ in 9gons &c

If z given t is found thus $t = \frac{1 + \sqrt{1 + 8z}}{2}$ in tri. $t = \frac{\sqrt{0 + 16z}}{4}$ in 4gons $t = \frac{+1 + \sqrt{1 + 24z}}{6}$, in 5gons. $t = \frac{+2 + \sqrt{4 + 32z}}{8}$ in 6gons &c. As if y^e side 12 of a tri given. y^e tri $= z = \frac{12 \times 12 + 12}{2} = 78$ &c & if $z = 21$ be octangled. $t = \frac{+4 + \sqrt{16 + 48z}}{12} = \frac{4 + \sqrt{16 + 48 \times 21}}{12}$

$t = \frac{4 + \sqrt{1024}}{12} = 3$.

July 4th 1699. By consulting an accompt of my expenses at Cambridge in ~~the~~ years 1663 & 1664 I find that in ye year 1664 a little before Christmas I being then senior Sophister, I ~~commenced~~ bought Schooten's Miscellanies & Cartes's Geometry (having read this Geometry & Oughtred's clavis above half a year before) & borrowed Wallis's works & by consequence made these Annotations out of Schooten & Wallis in winter between the years 1664 & 1665. At wch time I found the method of Infinite series. And in summer 1665 being forced from Cambridge by the Plague I computed ye area of ye Hyperbola at Boothby in Lincolnshire to two & fifty figures by the same method.

Is. Newton

Annotations out of Dr Wallis his Arithmetica infinitorum.

1 A primanary series of quantys is arithmetically proportionall, as 0, 1, 2, 3, 4. & its index is 1

A Secundanary series are those whose roots are arithmetically proportionall; as 0, 1, 4, 9, 16. & its index is 2

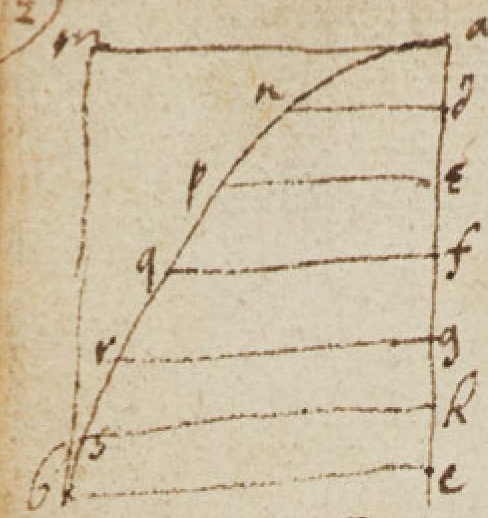
A Tertianary, quartanary, quintanary series of quantitys are those whose cube, squaresquare, squarecube roots are Arithmetically Proportionall as 0, 1, 8, 27, 64./ 0, 1, 16, 81, 156./ 0, 1, 32, 243, 624. &c Their indices being 3, 4, 5 &c.

3 Subsecundanary, subtertianary, series &c: are those whose squares, cubes, &c are arithmetically proportionall, as $\sqrt{0}$, $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$. $\sqrt{c:0}$, $\sqrt{c:1}$, $\sqrt{c:2}$, $\sqrt{c:3}$ &c. Theire indices being $\frac{1}{2}$, $\frac{1}{3}$, &c.

2 Primary Secundanary, tertianary series &c are said to bee reciprocally proportionall ([illegible] yt is to ye same [illegible] increasing) which continually decrease as. $\frac{1}{0}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. $\frac{1}{0}, \frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$. $\frac{1}{0}, \frac{1}{1}, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}$. Their indices being negative as -1, -2, -3. ~~&c~~

4 The indices of compound or mixt of rationall & irrationall series, by multiplying or dividing ye indices of ye simple series may bee found as in a subsecundanary progression cubed $\sqrt{0}$, $\sqrt{1}$, $\sqrt{8}$, $\sqrt{27}$, $\sqrt{64}$ ye index is $\frac{1}{2} \times 3 = \frac{3}{2}$. So in ye cube roots of a secundanary progression, $\sqrt{c:0}$, $\sqrt{c:1}$, $\sqrt{c:4}$, $\sqrt{c:9}$ &c. ye index is $\frac{1}{3} \times 2 = \frac{2}{3}$. So in irrationall reciprocall progressions $\sqrt{qq:\frac{1}{0}}$, $\sqrt{qq:\frac{1}{1}}$, $\sqrt{qq:\frac{1}{2}}$, $\sqrt{qq:\frac{1}{3}}$, &c ye index is $-1 \times \frac{1}{4} = \frac{-1}{4}$.

2)



Now suppose ye line ac be divided into an infinite number of equall pts ad, de, ef, fg &c, from each of wch are drawne ~~perpendiculars~~ parallells nd pe qf &c. wch ~~wch~~ increase continually in some of ye foregoing progressions or in some progression compounded of ym, all those lines may be taken for ye surface bqnac, & to know wt proportion ~~all those lines have~~ that superficies hath to ye superficies ambc yt is wt proportion all those lines have to soe many equall to ye greatest of ym, I say as ye index of ye progression increased by an unite is to an unite soe is ye square abcm to ye area of ye crooked line. As if abc is a parabola ye lines nd, pe, qf, &c: are a subsecundanary series (for $y=\sqrt{rx}$) whos index is $\frac{1}{2}$ wch added to an unite is $1+\frac{1}{2}=\frac{3}{2}$ Therefore $\frac{3}{2} : 1 :: 3 : 2$ so is ye square ambc to ye area of ye Parab. (ye names of ye lines are (ad), ae, af &c $= x$. dn, pe, qf &c $= y$. $ac=p$. $bc=q$.) The case is ye same if abc bee supposed a sollid, as suppose ~~its nature~~ a Parabolicall conoides. yn since ye nature of it is $rx=yy$. yy designes ye squares nd, pe, qf &c: all wch taken together are equivalent to ye sollid. & those □s increase in ye same proportion wch rx, or x doth yt is they are a primanary series whose index is 1 to wch (according to ye rule) ad an unite & tis 2. Therefor $1:2::$ soe are all ye □s of ye Primary series to soe many □s equal to ye greatest of yt series, & soe is ye conoides to a cilinder of ye same altitude.

Also if a superficies bee compounded of 2
or more of these series, Their area is as
easily found; as if y^e nature of y^e line bee
~~y = ...~~, $y = aa - xx$, or $y = a^4 - 2aaxx + x^4$
or $y = a^6 - 3a^4xx + 3aax^4 - x^6$. &c. Their
areas will bee to y^e parallelograms about them
as 2 to 3, as 8 to 15, as 48 to 105 &c.
But if I put in y^e intermediate termes in
these last named lines their order will bee
$y = \sqrt{aa - xx}$, $y = aa - xx$, $y = \overline{aa - xx}\sqrt{aa - xx}$,
$y = a^4 - 2aaxx + x^4$. $y = \overline{a^4 - 2aaxx + x^4}\sqrt{aa - xx}$.
$y = a^6 - 3a^4xx + 3aax^4 - x^6$; &c: & since these
lines observe a geometricall progression their
areas must observe some kind of progression.
of wch every other terme is given viz

$1.\ \square.\ \frac{2}{3}.\ *.\ \frac{8}{15}.\ *.\ \frac{48}{105}.\ *.\ \frac{384}{945}.\ *.\ \frac{3840}{10395}.$

Twixt wch termes if y^e intermediate termes $\square.*$
can bee found y^e 2d $\square$ will give y^e area of
y^e line $y = \sqrt{aa - xx}$, y^e circle. Soe likewise in
this progression of lines $y = 1$. $y = \sqrt{ax - xx}$. $y = ax - xx$
$y = \overline{ax - xx}\sqrt{ax - xx}$. $y = aaxx - 2ax^3 + x^4$ &c:
y^e progression of their area's is $1 : \square : \frac{1}{6} : * :$
$\frac{1}{30} : * : \frac{1}{140} ; * \frac{1}{630}$:&c. y^e 2d terme if it can bee
found givs y^e area of y^e ⊙ for as its denominator
to its numerator so is y^e $\square$ of y^e diameter to y^e
area of a semicircle. If this last progression bee
multiplyed by y^e respective termes in y^e progression
1. 2. 3. 4 & it may bee diminished, y^e reslut being
$1.\ 2\square.\ \frac{1}{2}.\ 4*.\ \frac{1}{6}.\ 6*.\ \frac{1}{20}.\ 8*.\ \frac{1}{70}$ soe y^t in this
progression $1,\ b,\ \frac{1}{2},\ c,\ \frac{1}{6},\ d,\ \frac{1}{20},\ e,\ \frac{1}{70},\ f$, &c: if

b can bee found y^n, ~~[illegible]~~ y^e $\square$ of y^e diameter to y^e area of y^e circle is as y^e denominator of b to its numerator. Likewise y^e 1st series of areas may bee diminished by multiplying each terme by its correspondent terme in this progression 1, 2, 3, 4, 5, 6, &c: & it will become, 1, a, 2, b, $\frac{8}{3}$, c, $\frac{48}{15}$, d, $\frac{384}{105}$, e, ~~[illegible]~~ $\frac{3840}{945}$. &c. In wch if a can bee found. y^n as y^e denom of a to its num: so y^e $\square$ of y^e Radius to a semicircle, y^t making y^e radius $= q$. $2aq = \odot$. The same kinds of changes may bee performed by ~~[illegible]~~ any other progressions, as by division by y^e geometricall progression 1, 2, 4, 8, 16, & y^e first series of areas becomes, 1, g, $\frac{1}{6}$, h, $\frac{1}{30}$, k, $\frac{1}{140}$, l, $\frac{1}{630}$, &c viz y^e same wth y^e 2d series. Also these changes may be done by addition or substraction of mutuall termes in 2 proportions. Soe y^t y^e most convenient way may bee chosen, whereby to reduce any series of proportions to y^e most convenent forme.

Now if it be propounded to find these middle termes, It will bee convenient to find how the given proportion may bee deduced from an Arithmeticall, Geometricall, or some other familiar proportion, viz whose meane termes may be found; as this progression $1 . \frac{2}{3} . \frac{8}{15} . \frac{48}{105}$ deduceth its originall from this $A \times \frac{0 \times 2 \times 4 \times 6 \times 8}{1 \times 3 \times 5 \times 7 \times 9}$ &c in wch A is an infinite number $= \frac{1}{0}$.

It will also be convenient to find what relatio all y^e other meanes have to y^e first soe y^t if y^e first bee had all y^e other may bee deduced thence. As in this case suppose y^e 1st meane to bee a. The progression will bee

$\frac{1}{2}a : 1 : a : \frac{3}{2} : \frac{4a}{3} : \frac{15}{8} : \frac{8a}{5} : \frac{105}{48} : \frac{64a}{35} : \frac{945}{384} : \frac{640a}{315}$

deducing its originall from $A \times \frac{0 \times 2 \times 4 \times 6 \times 8 \times 10}{1 \times 3 \times 5 \times 7 \times 9 \times 11}$

& from this $A(=\frac{1}{2}) \times \frac{2a \times 4a \times 6a \times 8a \times 10a}{1 \times 3 \times 5 \times 7 \times 9}$. &c 17

(note y^t y^e proportions of these ~~meane~~ termes to one another, or to (a), are found ~~f~~ by finding y^e proportion of y^e circle $y=\sqrt{aa-xx}$ to y^e line $y = aa-xx\sqrt{aa-xx}$ &c).

In this case to find y^e quantity a; ~~it may be considered y^t $\frac{a}{\frac{1}{2}a} = 2$.~~ Naming y^e termes in y^e progress: $\frac{1}{2}a : 1 : a : \frac{3}{2} : \frac{4a}{3} : \frac{15}{8} : \frac{8a}{5} : \frac{35}{16}$.

$b \quad c \quad d \quad e \quad f \quad g \quad h \quad k$.

1st observe y^t $\frac{d}{b} = 2$. $\frac{e}{c} = \frac{3}{2}$. $\frac{f}{d} = \frac{4}{3}$. $\frac{g}{e} = \frac{5}{4}$. $\frac{h}{f} = \frac{6}{5}$ &c y^e proportions still decreasing & therefore y^t in $\frac{c}{b} . \frac{d}{c} . \frac{e}{d} . \frac{f}{e} . \frac{g}{f} . \frac{h}{g} . \frac{k}{h}$ &c: y^e latter terme is lesse y^n y^e former; & therefore $\frac{dd}{ee}$ is $\begin{cases} \text{less } y^n\ \frac{d}{c} \times \frac{c}{b} = \frac{d}{b} = 2. \\ \text{greater } y^n\ \frac{d}{c} \times \frac{e}{d} = \frac{e}{c} = \frac{3}{2} \end{cases}$

or $\frac{d}{1}$ is $\begin{cases} \text{less } y^n\ 1 \times \sqrt{2} = \sqrt{1+\frac{1}{1}} \\ \text{greater } y^n\ 1 \times \sqrt{\frac{3}{2}} = \sqrt{1+\frac{1}{2}} \end{cases}$. Also

$\frac{ff}{ee}$ is $\begin{cases} \text{lesse } y^n\ \frac{f}{e} \times \frac{e}{d} = \frac{f}{d} = \frac{4}{3} \\ \text{greater } y^n\ \frac{f}{e} \times \frac{g}{f} = \frac{g}{e} = \frac{5}{4} \end{cases} = l : \frac{8a}{9} = \frac{2 \times 4a}{3 \times 3}$.

~~Therefore $3 \times 3 \sqrt{\frac{4}{3}}$~~
Therefore a is $\begin{cases} \text{lesse } y^n\ \frac{3 \times 3}{2 \times 4}\sqrt{\frac{4}{3}} \\ \text{greater } y^n\ \frac{3 \times 3}{2 \times 4}\sqrt{\frac{5}{4}} \end{cases}$. And so by y^e same reasoning.

a is $\begin{cases} \text{less } y^n\ \frac{9 \times 25 \times 49 \times 81 \times 121 \times 169}{2 \times 16 \times 36 \times 64 \times 100 \times 144 \times 14}\sqrt{\frac{14}{13}}. \\ \text{greater } y^n\ \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 9 \times 9 \times 11 \times 11 \times 13 \times 13}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times 8 \times 10 \times 10 \times 12 \times 12 \times 14}\sqrt{\frac{15}{14}}. \end{cases}$ &c.

Thus Wallis doth it, but it may bee done thus.

a is $\begin{cases} \text{greater } y^n\ 1. \\ \text{lesse than } \frac{3}{2}. \end{cases}$ | $\frac{4a}{3}$ is $\begin{cases} \text{greater than } \frac{3}{2} \\ \text{lesse then } \frac{15}{8} \end{cases}$ & therefore

a is $\begin{cases} \text{greater } y^n \ \dfrac{3\times3}{2\times4} \\ \text{lesse then } \dfrac{3\times3\times5}{2\times4\times4} \end{cases}$. $\dfrac{8a}{5}$ is $\begin{cases} \text{greater } y^n \ \dfrac{15}{8} \\ \text{lesse } y^n \ \dfrac{35}{16} \end{cases}$

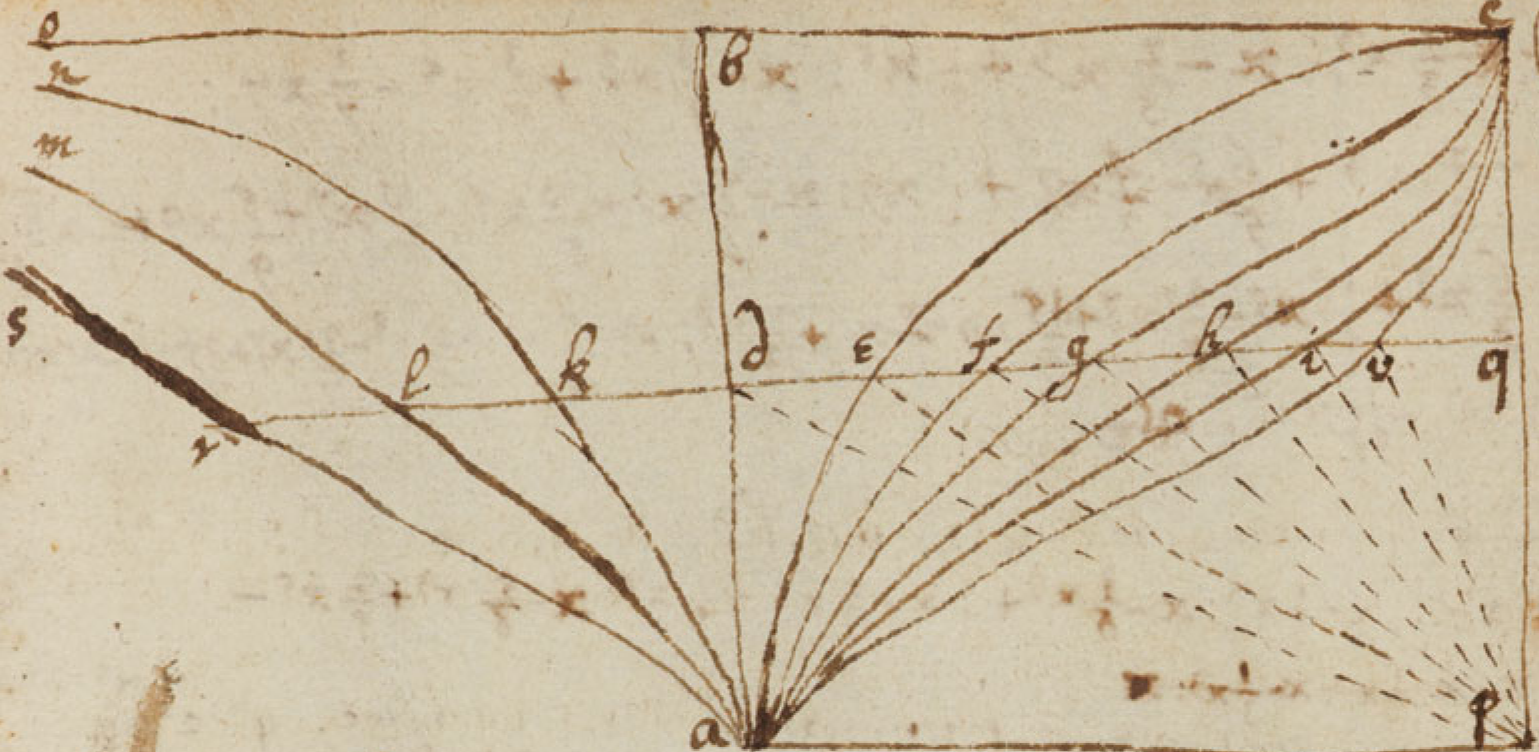
y^t is a is $\begin{cases} \text{greater then } \dfrac{3\times3\times5\times5}{2\times4\times4\times6} = \dfrac{3\times5\times5}{2\times4\times4\times2} \\ \text{lesse then } \dfrac{3\times3\times5\times5\times7}{2\times4\times4\times6\times6} = \dfrac{5\times5\times7}{2\times4\times4\times4} \end{cases}$ &c.

By y^e same reasoning

a is $\begin{cases} \text{greater } y^n \ \dfrac{3}{2}\times\dfrac{3}{4}\times\dfrac{5}{4}\times\dfrac{5}{6}\times\dfrac{7}{6}\times\dfrac{7}{8}\times\dfrac{9}{8}\times\dfrac{9}{10} = \dfrac{5\times49\times81}{2\times16\times4\times64\times2}. \\ \text{lesse } y^n \ \dfrac{3}{2}\times\dfrac{3}{4}\times\dfrac{5}{4}\times\dfrac{5}{6}\times\dfrac{7}{6}\times\dfrac{7}{8}\times\dfrac{9}{8}\times\dfrac{9}{10}\times\dfrac{11}{10} = \dfrac{49\times81\times11}{2\times16\times4\times64\times4}. \end{cases}$

Or a is $\begin{cases} \text{greater } y^n \ \dfrac{11\times169\times225\times289\times369\times441}{2\times16\times4\times64\times4\times144\times4\times256\times4\times400\times2} \\ \text{lesse } y^n \ \dfrac{169\times225\times289\times369\times441\times23}{2\times16\times4\times64\times4\times144\times4\times256\times4\times400\times4}. \end{cases}$

Note y^t a is greater y^n $\frac{1}{2}$ these two summes.



Having y^e signe of any angle to find y^e angle or to find y^e content of any segm^{nt} of a circle

Suppose y^e circle to be aec its segm^t diameter $ap=pc=1$. y^e given sine $pq=x$, viz: y^e signe of y^e angle epa. y^e segment sought eapq. abcp the □ of its Radius. & y^e $qi : qh : qg : qf : qe : qd : qk : ql : qr$ &c are continually proportionall. Then is $eq=\sqrt{1-xx}$. $fq=1-xx$. $gq=\sqrt{1-xx \text{ in } \sqrt{1-xx}}$. $hq=1-2xx+x^4$. $iq=1-2x^2+x^4\sqrt{1-xx}$. $dq=1$. $kq=\frac{1}{\sqrt{1-xx}}$. $lq=\frac{1}{1-xx}$. $rq=\frac{1}{1-xx \text{ in } \sqrt{1-xx}}$. &c & since all these ordinately applyed lines in these figures abcq, aecq, afcq, agcq &c are geometrically proportionall their areas adqp, aeqp, afqp, agqp, ahpq &c will observe some proportion amongst one another. To find w^{ch} proportion, 1st $adqp = 1 \times x = x$. 2dly afc is a parab: therefore $afqp = x - \frac{x^3}{3}$. also since tis ~ $qh=1-2xx+x^4$, therefore $ahqp = x - \frac{2}{3}x^3 + \frac{1}{5}x^5$. Also $vq = 1-3xx+3x^4-x^6$, therefore $avqp = \frac{}{x-x^3} + \frac{3}{5}x^5 - \frac{1}{7}x^7$. & by the same proceeding y^e proportion may bee still continued after this manner

$x : x - \frac{1}{3}x^3 : x - \frac{2}{3}x^3 + \frac{1}{5}x^5 : x - \frac{3}{3}x^3 + \frac{3}{5}x^5 - \frac{1}{7}x^7 :$

$x - \frac{4}{3}x^3 + \frac{6}{5}x^5 - \frac{4}{7}x^7 + \frac{1}{9}x^9 : x - \frac{5}{3}x^3 + \frac{10}{5}x^5 - \frac{10}{7}x^7 + \frac{5}{9}x^9 - \frac{1}{11}x^{11} :$

$x - \frac{6}{3}x^3 + \frac{15}{5}x^5 - \frac{20}{7}x^7 + \frac{15}{9}x^9 - \frac{6}{11}x^{11} + \frac{1}{13}x^{13} ; x - \frac{7}{3}x^3 + \frac{21}{5}x^5 - \frac{35}{7}x^7 + \frac{35}{9}x^9 - \frac{21}{11}x^{11}$

$+ \frac{7}{13}x^{13} - \frac{1}{15}x^{15}$. &c.

And if y^e meane termes be inserted it will be

$x : x - : x - \frac{1}{3}x^3 : x - \frac{3}{6}x^3 + : x - \frac{2}{3}x^3 + \frac{1}{5}x^5 : x - \frac{5}{6}x^3 + \frac{2}{5}x^5 -$

~~x : x - 1/6 x^3 : x - 1/3 x^3 : x~~

The first letters x run in this progression 1.1.1.1.1. &c. y^e 2^d x^3 in this $-\frac{1}{3} . \frac{0}{3} . \frac{1}{3} . \frac{2}{3} . \frac{3}{3} . \frac{4}{3} . \frac{5}{3}$ &c y^e 3^d x^5 in this ~~0+1 6.3.3.1.0.~~ 0+1=1. 1+2=3 3+3=6. 6+4=10. 10+5=15. y^e 4th ~~x this~~

1^st. $+x \times$ 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. &c

2^d. $-\frac{x^3}{3} \times$ 0. 0+1=1. 1+1=2. 2+1=3. 3+1=4. 4+1=5. 6. 7. 8. 9. 10. 11

3^d. $+\frac{x^5}{5} \times$ 0. 0+0=0. 0+1=1. 1+2=3. 3+3=6. 6+4=10. 15. 21. 28. 36. 45.

4^th. $-\frac{x^7}{7} \times$ 0. 0+0=0. 0+0=0. 0+1=1. 1+3=4. 4+6=10. 20. 35. 56. 84. 120.

5. $+\frac{x^9}{9} \times$ 0. 0+0=0. 0+0=0. 0+0=0. 0+1=1. 1+4=5. 15. 35. 70. 126. 210.

6. $-\frac{x^{11}}{11} \times$ 0. 0+0=0. 0+0=0. 0+0=0. 0+0=0. 0+1=1. 6. 21. 56. 126. 252.

1^st * 2^d. * 3^d * 4^th * 5^t * 6^t * 1. 7. 28. 84. 210.

* 7^th 1. 8. 36. 120.

* 8^th 1. 9. 45.

* 9^th 1. 10.

* 10^th 1

* 11^th.

Now if the meane termes in these progressions can bee calculated y^e first of y^m gives y^e area aeqp.

Which is thus done

1^st $+x \times$ 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.

2^d. $-\frac{x^3}{3} \times$ 0. $\frac{1}{2}$. 1. $\frac{3}{2}$. 2. $\frac{5}{2}$. 3. $\frac{7}{2}$. 4. $\frac{9}{2}$. 5. $\frac{11}{2}$. 6.

3^d. $+\frac{1}{5}x^5 \times$ 0. $-\frac{1}{8}$. 0. $\frac{3}{8}$. 1. $\frac{15}{8}$. 3. $\frac{35}{8}$. 6. $\frac{63}{8}$. 10. $\frac{99}{8}$. 15.

4. $-\frac{1}{7}x^7 \times$ 0. $+\frac{1}{16}$. 0. $-\frac{1}{16}$. 0. $\frac{5}{16}$. 1. $\frac{35}{16}$. 4. $\frac{105}{16}$. 10. $\frac{231}{16}$. 20

5. $+\frac{1}{9}x^9 \times$ 0. $-\frac{5}{128}$. 0. $\frac{3}{128}$. 0. $-\frac{5}{128}$. 0. $\frac{35}{128}$. 1 $\frac{315}{128}$. 5. $\frac{1155}{128}$. 35

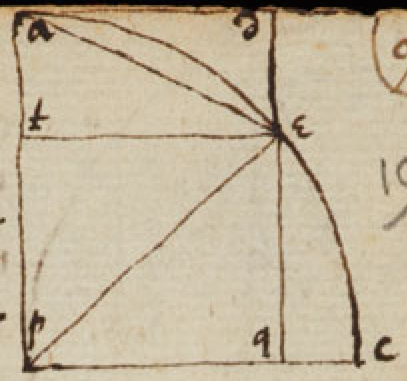
6. $-\frac{1}{11}x^{11} \times$ 0. $\frac{7}{256}$. 0. $\frac{-3}{256}$. 0. $\frac{3}{256}$. 0. $\frac{-7}{256}$. 0. $\frac{63}{256}$. 1. $\frac{693}{256}$. 21.

7 $\frac{1}{13}x^{13} \times$ 0. $\frac{-21}{1024}$. 0. $\frac{7}{1024}$. 0. $\frac{-5}{1024}$. 0. $\frac{[illegible]7}{1024}$. 0. $\frac{-21}{1024}$. 0. $\frac{231}{1024}$. 7. $\frac{3003}{1024}$

Soe y^t $1 \times x - \frac{1}{2} \times \frac{1}{3} x^3 - \frac{1}{8} \times \frac{1}{5} x^5 - \frac{1}{16} \times \frac{1}{7} x^7 - \frac{5}{128} \times \frac{1}{9} x^9$ &c. is y^e area, apq^e

y^t is $\frac{0}{0} x - \frac{0}{0} \times \frac{1}{2} \times \frac{1}{3} x^3 - \frac{0}{0} \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{5} x^5 - \frac{0}{0} \times \frac{1}{2} \times \frac{1}{4} \times \frac{3}{6} \times \frac{1}{7} x^7 - \frac{1 \times 3 \times 5\ x^9}{2 \times 4 \times 6 \times 8 \times 9}$ &c.

The progression may be deduced from hence $\frac{0 \times 1 \times -1 \times 3 \times 5 \times 7 \times -9 \times 11}{0 \times 2 \times 4 \times 6 \times 8 \times 10 \times 12 \times 14}$ &c.



Soe y^t if y^e given sine bee $pq=te=x$. & if y^e Radius $pc=1$. Then is y^e superficies $ape=x-x\sqrt{1-xx}-\frac{1}{6}x^3-\frac{1}{40}x^5-\frac{x^7}{112}-\frac{5x^9}{1152}-\frac{7x^{11}}{2816}$ &c: And y^e area $ade=\frac{x^3}{6}+\frac{x^5}{40}+\frac{x^7}{112}+\frac{5x^9}{1152}+\frac{7x^{11}}{2816}+\frac{21x^{13}}{13312}+\frac{11x^{15}}{10240}+\frac{429x^{17}}{557056}+\frac{715x^{19}}{1245184}+\frac{2431x^{21}}{5505024}$ &c. By w^ch meanes y^e angle ape is easily found for $aecpa:\angle apc=90::ape:\angle ape$.

The same may bee thus done.

$adp=\frac{x}{2}$. Or $2adp=x$. $2afp=x+\frac{x^3}{3}$. $2ahp=x+\frac{2x^3}{3}-\frac{3x^5}{5}$. And $2avp=x+x^3-\frac{9x^5}{5}+\frac{5}{7}x^7$. &c. as in this order

x. $x+\frac{1}{3}x^3$. $x+\frac{2}{3}x^3-\frac{3}{5}x^5$. $x+x^3-\frac{9}{5}x^5+\frac{5x^7}{7}$. $x+\frac{4x^3}{3}-\frac{18x^5}{5}+\frac{20x^7}{7}-\frac{7}{9}x^9$. $x+\frac{5x^3}{3}-\frac{30x^5}{5}+\frac{50x^7}{7}-\frac{35x^9}{9}+\frac{9x^{11}}{11}$. $x+\frac{6x^3}{3}-\frac{45x^5}{5}+\frac{100}{7}x^7-\frac{105x^9}{9}+\frac{54x^{11}}{11}-\frac{11x^{13}}{13}$. $x+\frac{7x^3}{3}-\frac{63x^5}{5}+\frac{175x^7}{7}-\frac{245x^9}{9}+\frac{189}{11}x^{11}-\frac{77x^{13}}{13}+\frac{13x^{15}}{15}$. &c

Which progressions w^th their intermediate termes ~~may~~ bee thus exhibited.

	$2adp=$	$2aep=$	$2afp=$	$2agp=$	$2ahp=$	$2aip=$	$2avp=$
$+x$ in	1.	1.	1.	1.	1.	1.	1.
$+\frac{x^3}{3}$ in	0.	$\frac{1}{2}$.	1.	$\frac{3}{2}$.	2.	$\frac{5}{2}$.	3.
$-\frac{3}{5}x^5$ in	0.	$-\frac{1}{8}$.	0.	$\frac{3}{8}$.	1.	$\frac{15}{8}$.	3.
$+\frac{5}{7}x^7$ in	0.	$\frac{1}{16}$.	0.	$-\frac{1}{16}$.	0.	$\frac{5}{16}$.	1.
$-\frac{7}{9}x^9$ in	0.	$-\frac{5}{128}$.	0.	$\frac{3}{128}$.	0.	$-\frac{5}{128}$.	0.
$+\frac{9}{11}x^{11}$ in	0.	$\frac{7}{256}$.	0.	$-\frac{3}{256}$.	0.	$\frac{3}{256}$.	0.

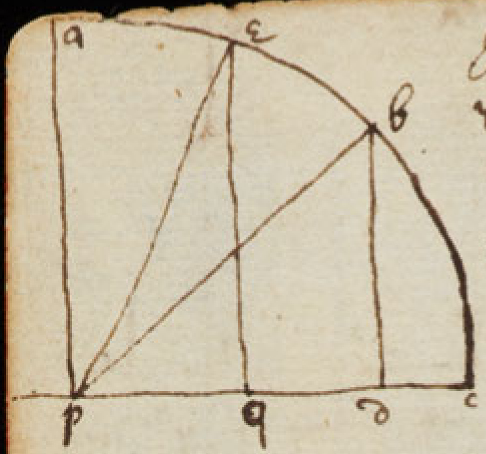
By w^ch it may appeare y^t if $pe=1$. $pq=x$. y^n $aep=\frac{1}{2}x+\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{2304}$ &c.

And y^e area aep given gives y^e angle ape for $apce:\angle apc=90::ape:\angle ape$. Likewise y^e angle ape given its sign may bee found hereby

Note y^t $\sqrt{1-xx}=\frac{x}{2}-\frac{3x^3}{8}-\frac{5x^5}{80}-\frac{7x^7}{224}-\frac{45x^9}{2304}-\frac{77x^{11}}{5632}$ &c that is $\sqrt{1-xx}=\frac{x}{2}-\frac{x^3}{4}-\frac{x^5}{16}-\frac{x^7}{32}-\frac{5x^9}{256}-\frac{7x^{11}}{512}-\frac{21x^{13}}{2048}$ &c. According to this progression $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{4}\times\frac{3\times5\times7\times9\times11\times13\times15\times17}{6\times8\times10\times12\times14\times16\times18\times20}$ &c.

Note also y^t y^e segment $ae=\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{2304}$. &c.

$aep=\frac{x}{2}+\frac{x^3}{12}+\frac{3x^5}{80}+\frac{5x^7}{224}+\frac{35x^9}{3304}+\frac{63x^{11}}{5632}+\frac{231x^{13}}{26624}+\frac{143x^{15}}{20480}+\frac{6435x^{17}}{1114112}+\frac{12155x^{19}}{2490368}+\frac{46189x^{21}}{11010048}$.



If $pq = a.\ qd = x.\ pc = 1 = pb.\ db = \sqrt{1-aa-2ax-xx}$

Then the areas of the lines in this progression.

$\cancel{1.\ \sqrt{1-aa-2ax-xx}.\ 1-aa-2ax-xx.\ \overline{1-aa-2ax-xx}\}^{\frac{3}{2}}.}$

$\cancel{1-2aa-4ax-2xx+a4+4a3x+6aaxx+4ax3+x4}.\ *.$

$\cancel{1-3aa+3a4+8a3x+8aaxx-a6}$ (supposing also $1-aa=b$.) viz: $1.\ \sqrt{b-2ax-xx}.\ b-2ax-xx.$

$\overline{b-2ax-xx}\}^{\frac{3}{2}}.\ bb-4abx\begin{smallmatrix}-2bxx\\+4aa\end{smallmatrix}+4ax3+x4.\ *$

$b3-6abbx-3bbxx+8abx3+3bx4-6ax5-x6.$ &c

$+12aabx^2 \pm 8a3x3 - 12aax4$

$+4abx3$

To Square the Hyperbola.

~~Epitome Geometriæ~~

So if nadm is an Hyperbola. & $ep=1=pa$. $pq=x$. qd, qe, qf, qg &c $=y$. & $\frac{1}{1+x}=y=dq$. $1=y=eq$. $1+x=y=qf$. $1+2x+xx=y=qg$. $1+3x+3xx+x^3$. $1+4x+6x^2+4x^3+x^4$. &c. There squares are. $*$. x. $x+\frac{xx}{2}$. $x+\frac{2xx}{2}+\frac{x^3}{3}$. $x+\frac{3xx}{2}+\frac{3x^3}{3}+\frac{x^4}{4}$. $x+\frac{4xx}{2}+\frac{6x^3}{3}+\frac{4x^4}{4}+\frac{x^5}{5}$. $x+\frac{5xx}{2}+\frac{10x^3}{3}+\frac{10x^4}{4}+\frac{5x^5}{5}+\frac{x^6}{6}$. &c As in ye following table, By whose first terme is represented ye square of ye Hyperbola, viz: yt it is

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\frac{x^7}{7}-\frac{x^8}{8}+\frac{x^9}{9}-\frac{x^{10}}{10}.\ \&c.$$

	adpq=	aepq=	afpq=	agpq=					
$x\times$	1.	1.	1.	1.	1.	1.	1.	1.	1.
$\frac{x^2}{2}\times$	−1.	0.	1.	2.	3.	4.	5.	6.	7.
$\frac{x^3}{3}\times$	1.	0.	0.	1.	3.	6.	10.	15.	21
$\frac{x^4}{4}\times$	−1.	0.	0.	0.	1.	4.	10.	20.	35.
$\frac{x^5}{5}\times$	1.	0.	0.	0.	0.	1.	5.	15.	35
$\frac{x^6}{6}\times$	−1.	0.	0.	0.	0.	0.	1.	6.	21
$\frac{x^7}{7}\times$	1.	0.	0.	0.	0.	0.	0.	1.	7.

As if $x=\frac{1}{10}$. or $cq=\frac{11}{10}=1,1$. ye is dapq $=\frac{1}{10}-\frac{1}{200}+\frac{1}{3000}-\frac{1}{40000}+\frac{1}{500000}-\frac{1}{6000000}$ &c

ye is apqd $=0,10000.00000.000000$
$\frac{1}{3}x^3=+0,00033.33333.333333$
$\frac{1}{5}x^5=+0,00000.20000.000000$
$\frac{1}{7}x^7=+0,00000.00142.857142$
$\frac{1}{9}x^9=+0,00000.00001.111111$
$\frac{1}{11}x^{11}=+0,00000.00000.009090$
$\frac{1}{13}x^{13}=+0,00000.00000.000076$
$\frac{1}{15}x^{15}=+0,00000.00000.000000$

0,1003353477.310755 Summa

0000.00000.0000 0.
3333333333.3333 3.33333.33333.3
000 0.000000.0000 0.00000.00000.0
857 1.42857.1428 5.71428.57142.8
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9090.90909.0909 0.90909.09090.9
9230.76923.07692.30769.2307 6.92307.69230
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0000.52631.57894.73684.21052.63157.94736.8
0000.00476.19047.61904.76190.47619.04761.
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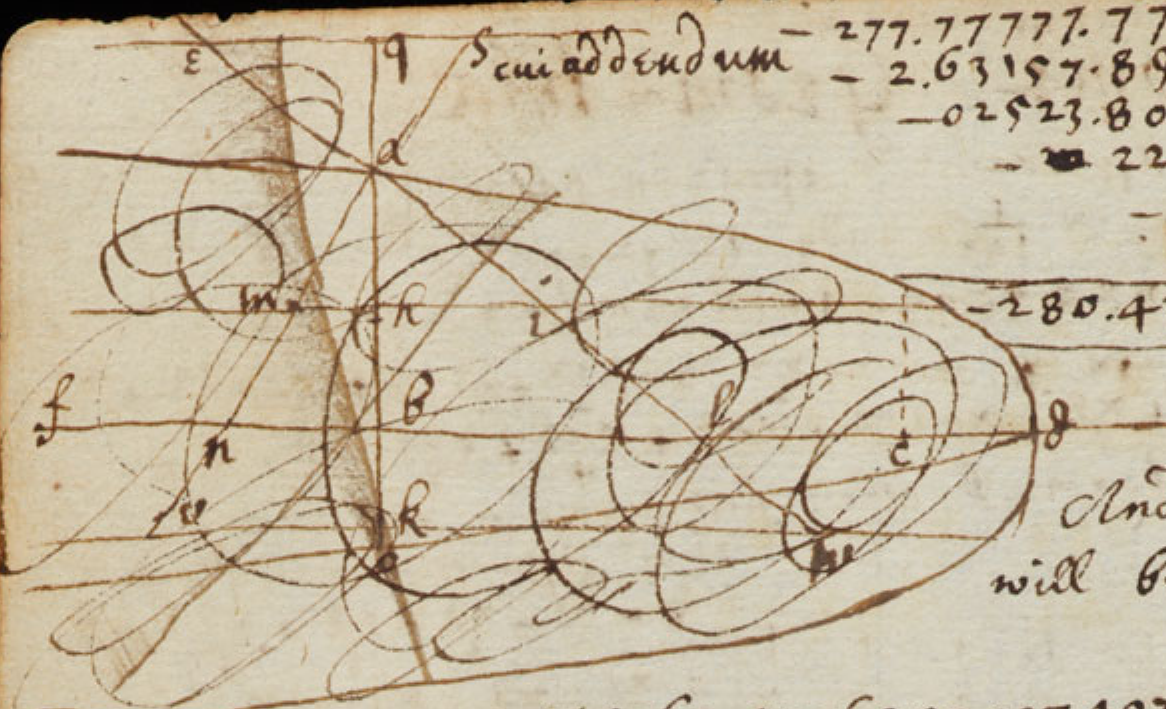
0000.00000.00000.03225.80645.16129.03225.80645.16129.03225.80
0000.00000.00000.00303.03030.30303.03030.30303.03030.30303.0
0000.00000.00000.00002.85714.28571.42857.14285.71428
0000.00000.00000.00000.27027.02702.70270.27027.02702.70270
0000.00000.00000.00000.00256.41025.64102.56410.25641.0
0000.00000.00000.00000.00002.43902.43902.43902.43902

$=8063.57265.52007.40736.63159.41506.3$ &c $=$ summa

0,00000.00000.00000.00000.00000.00000.00000.00000.0
−0,00500.00000.00000.00000.00000.00000.00000.00000.0
2.50000.00000.00000.00000.00000.00000.66666.66666.6
166.66666.66666.66666.66666.66666.66666.66666.0
12.50000.00000.00000.00000.00000.00000.00000.0
1000.00000.00000.00000.00000.00000.00000.0
83.33333.33333.33333.33333.33333.33333.3
7142.85714.28571.42857.14285.71428.5
630.55555.55555.55555.55555.55555.5
5045.45454.54545.45454.54545.45454.5
41666.66666.66666.66666.66666.6
384615.38461.53846.15384.61.5
3571428.57142.85714.28571.42857.1
33645.83333.33333.33333.3
29411.76470.58823.5

$-0.00502.51679.26750.72059.17744.28779.27385.30147.14044.12586.$



Semiaddendum − 277.77777.77777.7
− 2.63157.89421.0
− 02523.80952.4
− 22727.3
− 217.4
2.1

That is

−280.43459.71098.0

And so y^e summe will bee

+0,10033.53477.31075.58063.57265.52007.40736.63159.41506.3
−0.00502.51679.26750.72059.17144.28779.27385.30427.57503.8
0.09530.01798.04324.86004.40121.23228.13351.32731.84002.5

wch is y^e quantity of y^e area adpq, If cpab=1. & cp=ab=10pq & qd∥ap∥bc=ap.

In like manner if I make $x=\frac{1}{100}=pq$. The opperacon followeth.

0,01000.0,33333.33333.33333.33333.33333.33333.33333.33333.3
$+\frac{1}{5}x^5+\frac{1}{7}x^7=$ 20001.42857.14285.71428.57142.85714.28571.42857.14
~~11.12020.20202.02020.20202~~
$\frac{1}{9}x^9+\frac{1}{11}x^{11}=$ 11.11202.02020.20202.02020.20202.0
$\frac{1}{13}x^{13}+\frac{1}{15}x^{15}=$ 769.23076.92307.69230.7
$\frac{1}{17}x^{17}=$ 6666.66666.66666.6
$\frac{1}{19}x^{19}=$ 58823.52941.1
$\frac{1}{21}x^{21}=$ 5.26315.7
47.6

~~65.68993.81728.11594.~~
+0,01000.03333.53334.76201.58821.07551.40422.38870.97309.
58821.07551.40422.38870.97309.

−0,00005,00025,00166.66666.66666.66666.66666.66666.66666.6.
$-\frac{1}{8}x^8-\frac{1}{10}x^{10}-\frac{1}{12}x^{12}=$ −1250.10000.83333.33333.33333.33333.3
$-\frac{1}{14}x^{14}=$ −7.14285.71428.57142.8.
$-\frac{1}{16}x^{16}-\frac{1}{18}x^{18}$ −62.50555.55555.5
$-\frac{1}{20}x^{20}=$ − 5000.4

−0,00005,00025.00166.67916.76667.50007.14348.21984.17699.
67
+0,00995.03308.53168.08284.82153.57544.26074.16886.79610

Which is y^e quantity of y^e area apqd if 100p=cp. and abcp=1.

$y = db.\ x = ba$

$aay = x^3.\quad b + z = y\quad z = bc$

$aab + aaz = x^3.\quad b - z = y =\quad z = bf.$

$aab - aaz = x^3.\quad z - b = y =\quad z = bg$

$aaz - aab = x^3.\quad x = d + \xi.\ \xi = ak.$

$aay - d^3 - 3dd\xi - 3d\xi\xi - \xi^3 = 0$

$\left.\begin{matrix} aab + aaz \\ aab - aaz \\ aaz - aab \end{matrix}\right\} = d^3 + 3dd\xi + 3d\xi\xi + \xi^3.\quad x = b - \xi.\ aq = \xi$

$\left.\begin{matrix} aay \\ aab \pm aaz \\ aaz - aab \end{matrix}\right\} = d^3 - 3dd\xi + 3d\xi\xi - \xi^3.\quad x = \xi - b.\ ak = x$

$\left.\begin{matrix} aay \\ aab \pm aaz \\ aaz - aab \end{matrix}\right\} = \xi^3 - 3d\xi\xi + 3dd\xi - d^3.$

$na = \xi.\ nd = z.\quad \sqrt{c.\ \xi^3 - aaz + bb\xi}$

$ab : an :: a : b.$ & . $ab : nb :: a : c$ then ~~or if abc is right angle as~~

$\xi^3 = \frac{b^3}{a}z - \frac{bbc}{a}\xi.\quad \xi = d + \zeta.\ \zeta = mn$

$d^3 + 3dd\zeta + 3d\zeta\zeta + \zeta^3 - \frac{b^3}{a}z + \frac{bbcd}{a}$
$+ bbc\zeta$

$\partial b = x.\ ba = y.$

$aax = y^3$ ~~[illegible]~~. $x = b+z.\ z = bc.\quad y = c+\xi.\ \xi = ah$

$aab + aaz = \ \xi^3 + 3c\xi^2 + 3cc\xi + \xi^3.\quad y = c-\xi.\ \xi = aq$

$(x = b-z.\ z = bf.)$

$aab - aaz = \ c^3 - 3cc\xi + 3c\xi^2 \pm \xi^2.\quad y = \xi - c.\ \xi = ak.$

$(x = z-b.\ z = bg)$

$aaz - aab = \ \xi^3 - 3c\xi^2 + 3\xi c^2 - c^3.$

$na = y.\ \partial n = x.\ ab : an :: a : b.$

$ab : nb :: a : c.$ & $y^3 = \frac{b^3}{a}x - \frac{bbc}{a}y.$ Or

$\partial^2 x = \varepsilon\varepsilon y + y^3.$

$(x = z+o.\ z = nc)\ \varepsilon\varepsilon y + y^3.\ (y = n+\xi.\ \xi = ma)$

$\partial^2 z + \partial^2 o = \ \varepsilon^2 n + \varepsilon\varepsilon\xi + n^3 + 3nn\xi + 3n\xi^2 + \xi^3.$

$(x = z-o.\ z = gn)\quad (y = n-\xi.\ \xi = as)$

$\partial\partial z - \partial\partial o = \ \varepsilon\varepsilon n - \varepsilon\varepsilon\xi + n^3 - 3nn\xi + 3n\xi^2 - \xi^3.$

$(x = o-z.\ z = nf)\quad (y = \xi - n.\ \xi = av.)$

$\partial\partial o - \partial\partial z = \ \varepsilon\varepsilon\xi - \varepsilon\varepsilon n + \xi^3 - 3n\xi^2 + 3\xi n^2 - n^3.$

$\partial\partial z$

$al = y.\ \partial l = x.\ ab : al :: a : b.$ &

$al : bl :: b : c.$ whence $y^3 = \frac{xb^3 + yb^2c}{a}$

Or ~~[illegible]~~. &c: as before onely varying ye signes at $\varepsilon\varepsilon n$ & $\varepsilon\varepsilon\xi$.

$y^3 - \varepsilon\varepsilon y = \partial\partial x$

$ao = y.\ \partial o = x.\ a : b :: b\partial : \partial o.\ b : c :: \partial o : ob.$ &

$\frac{a^3}{b}x = y^3 - \frac{3c}{b}xyy + \frac{3cc}{bb}xxy - \frac{c^3x^3}{bb}.$

Dr Wallis in a letter to Sr Kenelme Digby promiseth ye squaring of ye Hyperbola by finding a meane proportion twixt 1, & $\frac{5}{6}$ in the progression $1, \frac{5}{6}, \frac{31}{30}, \frac{209}{140}, \frac{1471}{630}, \frac{10625}{2772}$ &c.

The resolution of Cubick equations out of Dr Wallis in his dedication before Meibomius confuted. 23

suppose $x=\pm a\pm\varepsilon$. y^n $x^3=\pm a^3\pm 3aa\varepsilon\pm 3a\varepsilon\varepsilon\pm\varepsilon^3$.

or $x^3=+3a\varepsilon x\pm a^3\pm\varepsilon^3$. that is making $a^3+\varepsilon^3=q$.

~~y^n $x^3=3a\varepsilon x\pm\varepsilon^3$~~. & $3a\varepsilon=p$. y^n $x^3 * +px\pm q$.

~~Againe suppose $y=\pm b\pm e$. Then~~

~~y^3~~ Againe suppose $x=a-\varepsilon$. y^n $x^3=a^3-3aa\varepsilon+3a\varepsilon\varepsilon-\varepsilon^3$.

y^t is making $a^3-\varepsilon^3=\pm q$, & $3a\varepsilon=p$, y^n $x^3=-px\pm q$.

Then in the first of these $p=3a\varepsilon$. or $\frac{p}{3\varepsilon}=a$.

or $\frac{p^3}{27\varepsilon^3}=a^3=q-\varepsilon^3$. Therefore $\varepsilon^6=q\varepsilon^3-\frac{p^3}{27}$.

~~y^t~~ $\varepsilon^3=\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq-\frac{p^3}{27}}$. & by y^e same reason $a^3=\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq-\frac{p^3}{27}}$

where y^e irrationall quantitys have divers signes otherwise $a^3+\varepsilon^3=q$ would bee false. Soe that

$x=\pm a\pm\varepsilon=\pm\sqrt{c:\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq-\frac{1}{27}p^3}}\pm\sqrt{c:\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq-\frac{1}{27}p^3}}$.

is a rule for resolving y^e equation $x^3 * -p\pm q=0$, when it hath but one roote y^t is when it may be generated according to the supposition $x=\pm a\pm\varepsilon$. &c. By y^e same reason $x^3 * +px\pm q$. may be resolved by this rule $x=a-\varepsilon=\sqrt{c:\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq+\frac{1}{27}p^3}}-\sqrt{c:\frac{1}{2}q\pm\sqrt{\frac{1}{4}qq+\frac{1}{27}p^3}}$.

But here observe y^t Dr Wallis would Argue y^t since in the first of these two cases (sometimes (viz when y^e equation hath 3 reall rootes) y^e ~~[illegible]~~ rule faileth as if it were imposible for y^e equation to have rootes when y^t it hath, therefore y^e fault is in Algebra. ~~&~~ therefore when ~~Analysis~~ Analysis leads us to an impossibility wee ought not to conclude y^e thing absolutely imposible, untill wee have tryed all y^e ways y^t may bee.

But let me answer y^t y^e fault is not in y^e Analysis in this Example, but in his opperation. for when y^e Equation $x^3 * * +px\pm q=0$, hath 3 roots hee supposeth it to have but one roote viz $x=\pm a\pm\varepsilon$. but since y^e Equation cannot be then generated according to y^t supposition it is imposible it should be reloved by it.

In like manner hee fayeth y^t Algebra represen=~~tel~~teth a thing possible when is not so as in this example, in y^e Δ abc, make $ab=1$. $bc=2$ $ac=4$. Then to find $dc=x$, worke thus,

$ad=4-x$. $bd \times bd = 1-16+8x-x^2 = 4-x^2$, therefore $8x=19$. or $x=\frac{19}{8}$. In w^ch opperacon all things proceede as possible though they are not soe for ac is greater y^n $ab+bc$.

Yet I answer y^t if y^e opperation & conclusion be compared together y^e absurdity will appeare, for in y^e equation $bd \times bd = 4 - xx = 4 - \frac{361}{64} = \frac{256-361}{64}$ or $bd \times bd = \frac{-105}{8}$. But it is impossible y^t a □ number should be negative.

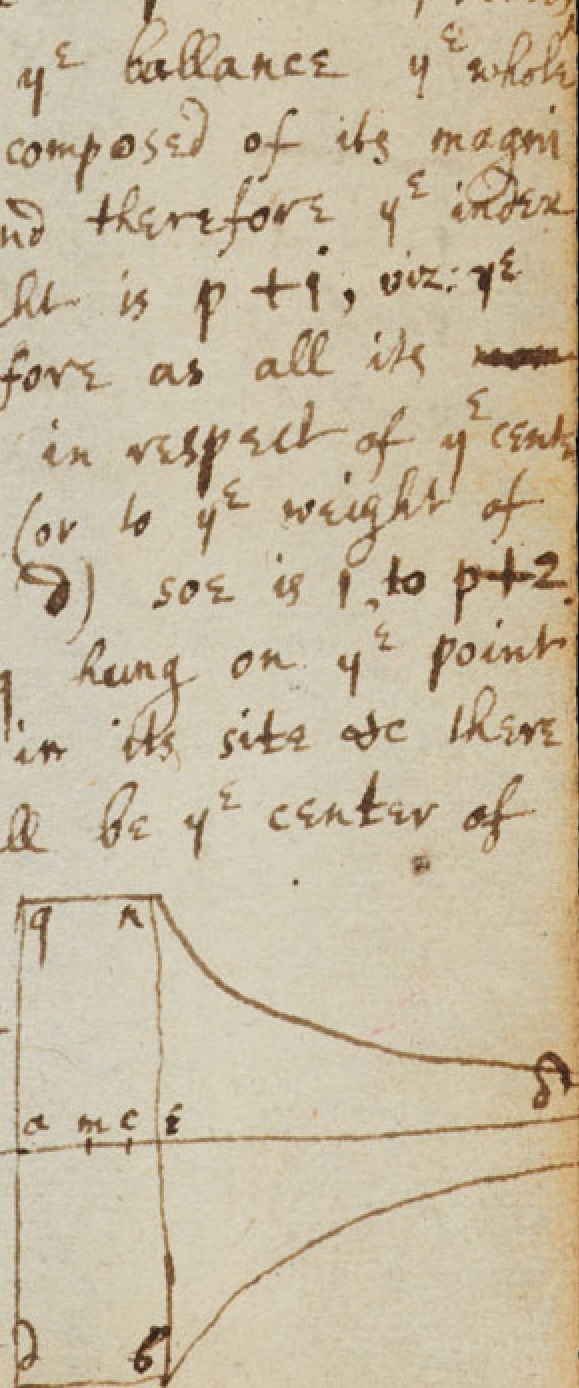
Thus $x=\sqrt{-b}$ is impossible. square it & tis $xx=-b$. Againe, & tis $x^4=bb$. Extract y^e root & tis $xx=b$. or $x=\sqrt{b}$. w^ch is possible. The reason of this ~~proceeding~~ Event is y^t $x^4-bb=0$ hath two possible rootes viz $x=\sqrt{b}$. $x=-\sqrt{b}$. & two impossible viz: $x=\sqrt{-b}$. $x=-\sqrt{-b}$.

Thus y^e valors of $x^8-a^8=0$ are $x=a, -a, \sqrt{-aa}, -\sqrt{-aa}, \sqrt{4:-a^4}, -\sqrt{4:-a^4}, \sqrt{-\sqrt{a^4}}, -\sqrt{-\sqrt{-a^4}}$.

Dr Wallis in a letter to Sr Kenelme Digby teacheth how to find ye center of gravity in divers lines first when their position is as in this figure, Suppose ad ye Axis, a their vertex ~~as 1 to ye series of progression cont.~~ Then

saying, as 1 to ye index of ye line increased by an unite (vide pag 2da) so cd to ca Then c is their center of gravity. The Demonstracon.

Let p bee ye index of ye series according to wch ye ordinately aplyed lines (parallell to db) increase, yn ~~p~~ 1 : p+1 :: area of ye line : to nmbq. ye distances of those ordinate lines from ye vertex a are equall to ye intercepted diameters & therefore a primanary series (whose index is 1) & since supposing a ye center of ye ballance ye whole weight of ye surface or figure is composed of its magnitude & distance from ye center and therefore ye index of all its moments or whole weight is p+1, viz: ye aggregate of ye other two. Therefore as all its moments (or ye weight of the figure in its site in respect of ye center a) are to soe many of ye greatest (or to ye weight of ye ▭ nmbq hung on ye point d) soe is 1, to p+2. and if ap : ad :: 1 : p+2, then nmbq hung on ye point q shall counterbalence ye figure in its site &c there fore if ac : cd :: p+1 : 1, c shall be ye center of gravity of those figures.



Also as the figure is now put extending infinitely towards δ if -2p+1 : -p+1 :: am : ac. ~~e~~ being ye center of qnbd yn c shall bee ye center of gravity of ye whole figure qndbδ.

Demonstration

since ye lines parallell to ad increase in series reciprocally proportionall their index is $-p$ & since ye halfes of those lines increas in ye same proportion their index is $-p$. whose extremitys or middle points of ye whole lines (supposing a ye center of ye ballance) are theire centers of gravity, their distances from a being proportionall to ye lines whose centers they are & consequently their index is $-p$ & since all ye moments (or whole weight of ye figure) increase in a proportion compounded of ye proportion of ye magnitudes & distances of ye lines from ye center a, they will be in a duplicate proportion of ye lines magnitudes that is a ~~series~~ reciprocall series whose index is $-2p$. There fore ye figure is ~~to~~ to ye inscribed parallelogram as 1 to $1-p$. & all its moments or whole weight of ~~the Para~~ in this its site to ~~the~~ weight of ye pgr as 1 to $1-2p$. Therefore if, am : ap :: $1-2p$: 1, the paralellogram hanging on ye point p shall counterballance ye whole figure in its site ade: whence ye point c may be found easily, ~~[illegible]~~ viz am : ac :: $1-2p$: $1-p$.

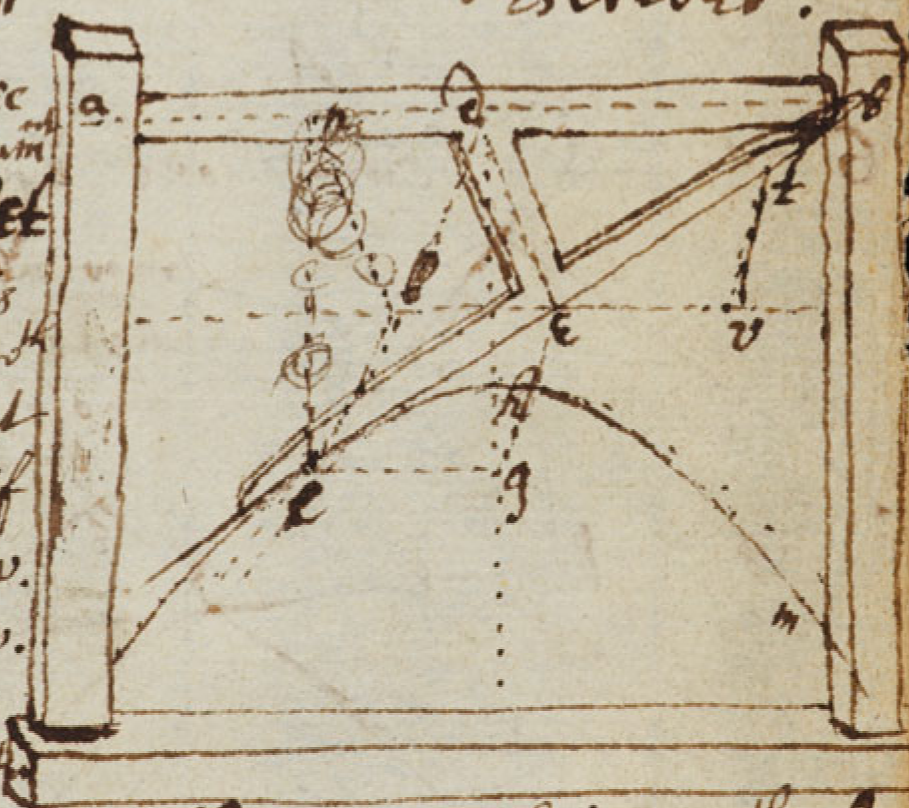
Of Refractions.

1 If ye ray ac bee refracted at the center of ye circle acdg towards d & $ab \perp be \perp gc \parallel ed$. Then suppose $ab : ed :: \partial : \varepsilon$. See Cartes Dioptricks

2 If there be an hyperbola the ~~whose~~ distance of whose foci are to its transverse axis kf ∂ to ε. Then ye ray $ac \parallel bd$ is refracted to ye exterior focus (d). See C: Diopt

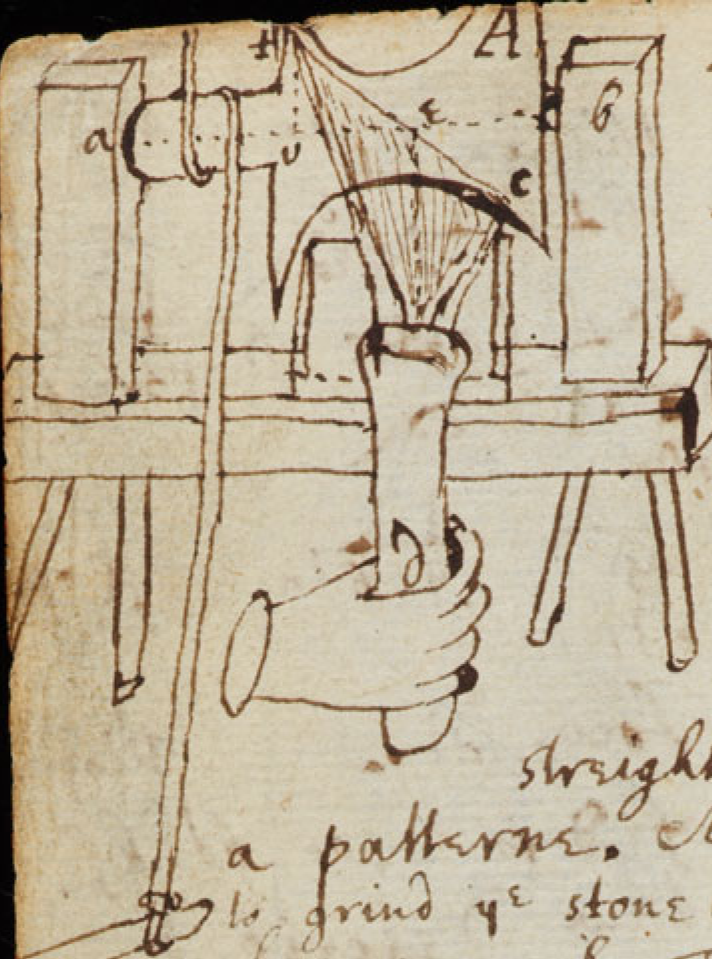
3 Having ye proportion of ∂ to ε, or $bd : kf$. The Hyperbola may bee thus described.

1 Upon ye centers a, b let ye instrument adbec bee moved in wch instrumt observe yt $ad \perp de \perp cbet$ & yt the beame cet is not in ye same plane wth adbe but intersects it at ye angle tev soe yt if $tv \perp ev$, then $\partial : \varepsilon :: et : tv$. Or $\partial : \varepsilon ::$ Rad: sine of $\angle tev$. Also make $de = \frac{q}{2}$, i.e. half ye transverse diamet. Then place the fiduciall side of ye plate ckm in the same plaine wth ab. & moving ye instrument adbec to & fro its edge cet shall cut or weare it into ye shape of ye desired Parabola. Or the plate ckm may bee filed away untill ye edg cet exactly touch it every where.



2 By the same proceeding Des-Cartes concave Hyperbolicall wheele may bee described by being turned wth a chissell ~~ttec~~ whose edge is a straight line inclined to the ~~axe~~ axis of the mandrill by ye $\angle tev$ wch angle is found by making $\partial : \varepsilon :: et : tv ::$ Rad: sine of $\angle tev$.

3 By the same reason a wheele may be turned Hyperbolically concave ye Hyperbola being convex. Or a Plate may bee turned Hyperbolically concav



4 Also Des=Cartes his Convex wheele B may bee turned or ground by a concave wheele A being made use of instead of a patterne

5 In turning the concave wheele A it will perhaps bee best to weare it with a stone p & let the straight edged chissell d serve for a patterne. And it may bee convenient to grind ye stone (or iron ~~tool~~) p into ye fashion of a cone S that it may fit ye hollow of the wheele A. The angle of wch cone being a right one or something greater it will almost grind the wheele to a true figure ~~the same done by~~ the shape of a Cone.

6 Draw $ab=q$; ac & cb of any length or intersecting one another at any angles. to make up the triangle abc. [Suppose yt ac bee called b, & that $e=qab$ then is d the distance of ye foci]. produce ac to d soe that

$ad=\frac{dd-ee}{b}$. Then draw bk through ye point d, & draw ek parallell to bc, lastly wth the sides ke, ek & angle hek describe the cone $hcklm$. Then produce ba to g &c indefinitely & ag being ye axis of a section that shall bee ye sought for Hyperbola

7 Since the proportion of cb to ab & $\angle cba$ is not determined it will bee most convenient to make

$cb = ab = \varepsilon = q$: & ~~$cb \perp ab \perp ak$~~. And than there will bee little danger of error at y^e^ vertex of the Hyperbola. And y^e^ calculation is readier for drawing $bp \perp ca$, Then is $cp = bp = ap = \frac{\varepsilon}{\sqrt{2}} = \sqrt{\frac{\varepsilon\varepsilon}{2}}$ & $pd = \frac{\delta\delta}{\varepsilon\sqrt{2}}$. Soe that $\varepsilon\varepsilon : \delta\delta :: cp = bp : pd ::$ ~~Rad. tangent~~ :: Radius : Tangent of pbd soe y^t^ y^e^ $\angle cbk = hek$ is easily found.

9 Having such a cone smothly pollished within & without, by the helpe of a square set y^e^ plate perpendicular to one side hee the fiduciall edge being distant from y^e^ vertex the length of $ae = \frac{\varepsilon\delta\delta - \varepsilon^3}{\delta\delta + \varepsilon\varepsilon}$ & if y^e^ edge of y^e^ plaine every where touch the cone, tis trew.

10 The exact distance (ae) of y^e^ plate from the vertex of y^e^ cone neede not bee much regarded for that changeth onely the bignesse not y^e^ shape of y^e^ figure.

[By y^e^ broken lookinglasse I find glasse refraction, y^t^ $\delta : \varepsilon :: 43 : 28 :: 1000 : 651+ :: 1536 : 1000$. These are ~~insensib~~ almost insensibly different from truth $\delta : \varepsilon :: 20 : 13 :: 100 : 65 :: 153- : 1000$. Or $\delta : \varepsilon ::$ ~~[illegible]~~ $23 : 15 :: 100 : 652+$ Or $\delta : \varepsilon :: 66 : 43 :: 100 : 651,5151+$. Or ~~$\delta : \varepsilon :: 100 : 651$~~.

ffor y^e^ Ellipsis $\frac{\delta\delta - \varepsilon\varepsilon}{\delta} x + \frac{\varepsilon\varepsilon - \delta\delta}{\delta\delta} xx = yy$

The former {descriptions / propositions} demonstrated.

Lemma. If in ye Opposite Hyperbolas ~~[illegible]~~ abc

edf (one of wch are to bee described) supposing $bd=d$. $kf=e$. $gk=x$. $gc=y$. $gc \perp gkd$. ~~& a point~~ & gc terminated by ye hyperbola

Then is $\frac{dd-ee}{ee}xx + \frac{dd-ee}{e}x = yy$. For

$bh=\frac{d-e}{2}$. $dh=\frac{d+e}{2}$. $bg=\frac{2x-d+e}{2}$. $gd=\frac{2x+d+e}{2}$.

$dc^2 = \frac{4xx+4dx+4ex+dd+2ed+ee+4yy}{4} = gd \times gd + gc \times gc$.

$bc^2 = \frac{4xx-4dx+4ex+dd-2ed+ee+4yy}{4} = gb^2 + gc^2$.

And since $dc = bc + kf$. Therefore $dc^2 = bc^2 + 2bc \times kf + kf^2$

$2dx+ed-ee = e\sqrt{4xx-4dx+4ex+dd-2ed+ee+4yy}$.

Both pts of wch □ed & ordered ye result is

$4ddxx-4eexx+4eddx-4e^3x-4eeyy=0$. That is

$\frac{dd-ee}{e}x + \frac{dd-ee}{ee}xx = yy$.

Desciption ye 1st demonstrated Synthetically. See yt Scheame

~~Naming ye quantitys $ed=\frac{e}{2}=dk$ [illegible] $gk=x$. $gc=y$.~~

Nameing ye quantitys, nk ed $= dk = \frac{e}{2}$. $gk=x$. $gc=bc=y$.

$dg = x + \frac{e}{2} = nc$. $cg \perp dkg$. therefore $ed^2 = x^2+ex+\frac{ee}{4}+y^2$.

$ce^2 = xx+ex+yy$. $eg^2 = xx+ex$. Also

$d : e :: et : tv :: ce : eg$, therefore $ddxx+ddex = eex^2+e^3x+e^2yy$

That is $\frac{dd-ee}{e}x + \frac{dd-ee}{ee}xx = yy$. As in ye lema.

The same demonstrated Analytically ~~Synthetically~~.

Nameing ye quantitys, $de = dk = a$. $gk=x$. $gc=y$.

$dg = a+x$. $dc^2 = aa+2ax+xx+yy$. $ce^2 = 2ax+x^2+yy$.

$eg^2 = xx+ex$. Supose yt ~~[illegible]~~ $b:c :: et : tv :: ce : eg$.

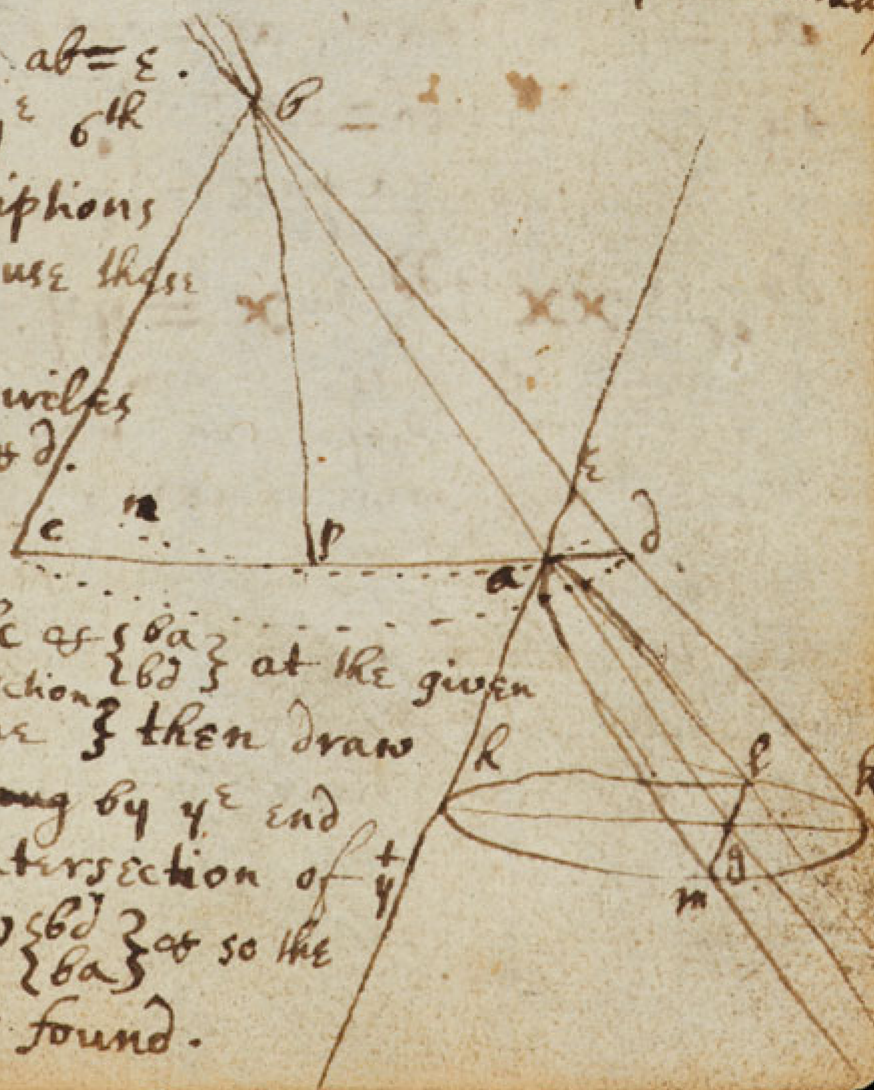
Then is $bbxx + bbex = 2ccax + ccxx + ccyy$.
That is $\frac{bb-cc}{cc}xx + \frac{bbe-2cca}{cc}x = yy$. Therefore ye line chm is a Conick Section & since (bb) is greater yn (cc) tis an Hyperbola, wch yt it may bee ye same wth yt in ye lemma, their correspondt termes are to bee compared together & soe I find yt $\frac{bb-cc}{cc}xx = \frac{dd-ee}{ee}xx$. & $\frac{bbe-2cca}{cc}x = \frac{dd-ee}{e}x$
by ye 1st =tion $bb = \frac{ccdd}{ee}$. or $b = \frac{cd}{e}$. yt is $b:c::d:e$.
by ye 2d $ccee - ccdd + bbee = 2ccea$. And by substituting $\frac{ccdd}{ee}$ into the place of bb, and ordering it tis ~~[illegible]~~ $ccee = 2ccea$. Or $\frac{e}{2} = a$. Therefore if I take $\frac{e}{2} = a = de$. & $d:e::b:c::et:tv$. then shall chm bee ye Hyperbola desired Q:E:D.

The 2d 3d 4th & 5th Propositions are manifest from this

~~The 6th Description demonstrated Synthetically~~

~~the quantity named are ab=e.~~

Instead of ye 6th & 7th Descriptions wch are false use these



6 Draw 2 concentrick circles (na & cd) wth ye Radii e & d. Then from ye comon center b draw 2 lines bc & {ba, bd} at the given angle {bae=abc, abd=cbd} of {section, ye Cone} then draw a line cad from c by ye end of ye Rad {ba, bd} & to ye intersection of yt line wth ye circle {cd, na} draw {bd, ba} & so the ∠ of {ye cone, kek=cbd, section, eab=abc} is found.

Or w^ch is the same ~~if...~~ make $ab=\varepsilon$. $bd=\partial$ & then if y^t cone is sought ~~make $cd=2v$~~ the $\angle cba$ being given, make $ac=a$. Then is $cd=\frac{\partial\partial-\varepsilon\varepsilon+aa}{a}$. & soe y^e $\angle cbc=a\varepsilon d$ is knowne & also $a\varepsilon=\varepsilon d=\frac{\partial^3-\partial\varepsilon\varepsilon}{\partial\partial-\varepsilon\varepsilon+aa}$, & $ad=\frac{\partial\partial-\varepsilon\varepsilon}{a}$. ~~Bf~~

But if y^e $\angle ba\varepsilon=abc$ of y^e section is sought y^e cone being given y^n make $cd=b$. And it will bee $ac=b+\sqrt{\varepsilon\varepsilon-\partial\partial+bb}$. & soe $\angle abc=ba\varepsilon$ is given also $ad=b-\sqrt{\varepsilon\varepsilon-\partial\partial+bb}$. & $a\varepsilon=\frac{\partial b-\partial\sqrt{\varepsilon\varepsilon-\partial\partial+bb}}{2b}$

In generall observe y^t in any cone cut any ways $bd=b\varepsilon+\varepsilon a=\partial$, & $ba=\varepsilon$.

7. Deslartes his wheele, thus described, cut by any plaine produceth one of y^e Conick=Sections.

Description y^e 6^th Demonstrated. Synthetically

Call, $bd=\partial$. $ba=\varepsilon$. $cp=pd=a$. $bp=\sqrt{\partial\partial-aa}$. $ag=x$. $ap=\sqrt{\varepsilon\varepsilon-\partial\partial+aa}$. $ac=a+\sqrt{\varepsilon\varepsilon-\partial\partial+aa}$. $ad=a-\sqrt{\varepsilon\varepsilon-\partial\partial+aa}$.

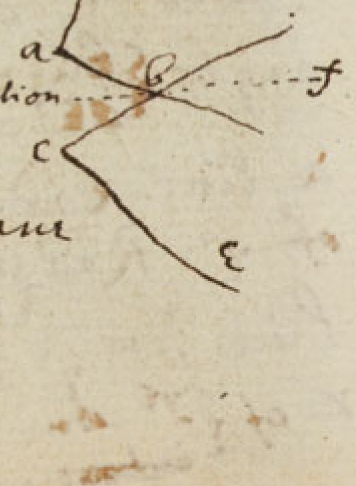
$ba:ac::ag:gh=\frac{ax+x\sqrt{\varepsilon\varepsilon-\partial\partial+aa}}{\varepsilon}$.

$ba:ad::bg:gk=\frac{\varepsilon a+ax}{\varepsilon}-\frac{\varepsilon-x}{\varepsilon}\sqrt{\varepsilon\varepsilon-\partial\partial+aa}$.

$gk\times gh=gm^2=y^2$. Therefore

$\frac{\partial\partial-\varepsilon\varepsilon}{\varepsilon\varepsilon}xx+\frac{\partial\partial-\varepsilon\varepsilon}{\varepsilon}x=yy$. by ordering y^e result of $(gk\times gk)$. w^ch is like y^t in the lemma.

The 7^th Proposition may be easyly demonstrated after the same manner.



If the two equall cones bad bcd intersect the one the other soe y^t $ab=bc$ their intersection (bf) shall bee one of y^e Conick sections as they had each beene intersected by the plane bf.

To describe y^e Parabola (& other figures after y^e same manner) pretty exactly.

Make a Squire cbe, soe y^t $cb=\frac{r}{2}$ (for then the circle described by (bc) will bee as crooked as y^e Parabola at the vertex d). Divide y^e other leg (be) of y^e Squire into any number of pts.

Then get a plate of Brasse &c: lkfd straight & even. And taking one point d for y^e vertex of it & another point c for y^e Squire to move on soe y^t $cd=cb=\frac{r}{2}$, & wearing away y^e edge of the plate untill (y^e Squire being erected) $ab=qd$. the squire touching y^e plate at a. thus shall y^e edge adf become Parabolicall. ~~By~~ y^e Rad: ab describe a circle ~~adg & by that meanes it~~ may bee knowne when $ab=al$. Instead of y^e leg be a circle may be used.

~~Demonstraco~~

Suppose $aq=y$. $qd=x$. $cd=eb=\frac{r}{2}$. Then is $ad=\sqrt{xx+yy}=ab$. & $ac=\sqrt{\frac{rr}{4}+xx+yy}$. And $cq=\sqrt{\frac{rr}{4}+xx}$. & $dq=x$.

Demonstracon.

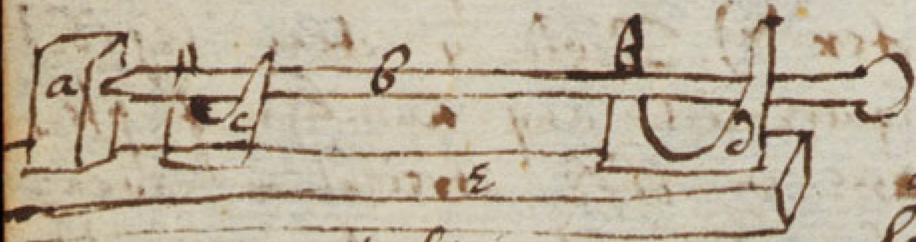
$qd=x$. $cd=\frac{r}{2}=cb$. $cq=\frac{r}{2}-x$. $aq=\sqrt{rx}=y$. $ac^2==\frac{rr}{4}+xx$

$ab^2=\frac{rr}{4}+xx-\frac{rr}{4}$ & $ab=x$. Q.E.D.

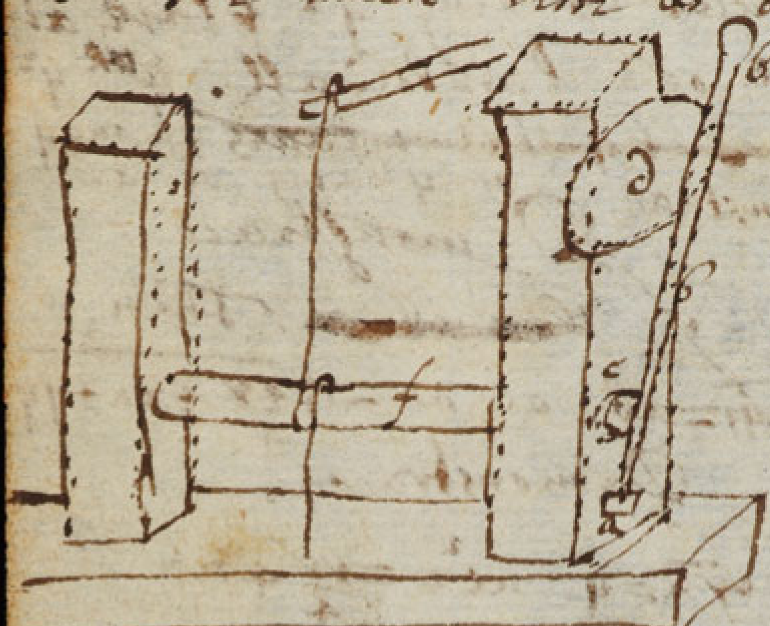
Another description of y^e Parabola w^{th} y^e compasses. Make $ab=bc=\frac{r}{4}$. Make $ce=cd$ & $ce\perp bd$. Make $af=ae$, & $bf=bd$ then shall f bee a point in y^e Parabola.

Another. Make $ab=\frac{r+x}{2}=ac$. $eb=x\perp ce$ & y^e point c shall bee in y^e parabola. This like y^e first by calculation may bee made use of in other lines.

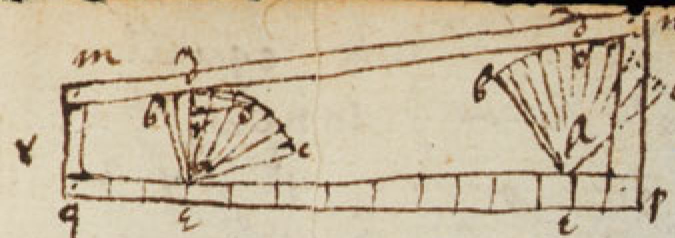
The manner wherby any kind of little lines may be described very accurately. Also that the same Instrument serve for all lines (though never so small) differing in quantity but not in quality.



Make ye plate d of ye figure required (by some of ye former meanes) the larger the better. Then hold the straight steele staffe b against the center a & roule it to & fro it shall grind c into ye same figure but soe much lesse as ac is lesse yn ad.

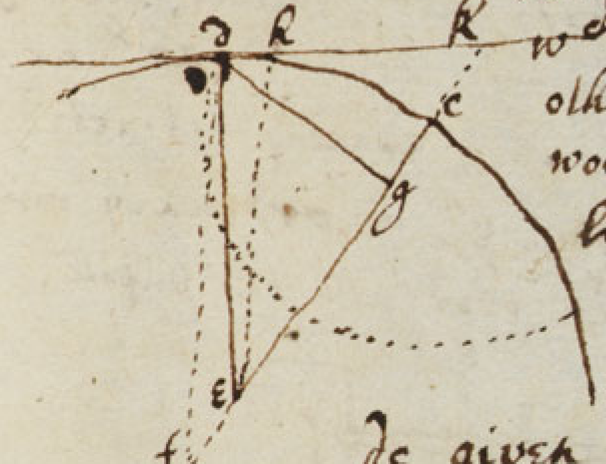


Soe if ye glass c bee fastened upon ye mandrill f, it may be ground acording to ye sollid ~~fig~~ figure d by ye helpe of a stick of steele (as a ~~cilind~~ cone) whose cuspis is in ye hole a upon wch it is moved as on a center. when ye ~~stick~~ cone b leanes uppon ye vertices of d & c it must be perpendicular to the mandrill f. Perhaps it may be convenient to cause ye cone b to turne about its axis. Or it may bee better instead of ye nutt at a wth a hole in it to make a sharpe pointed nutt, & instead of ye ~~cilinder~~ cone b to make use of a broad plate to cover a, c & d & move every way upon them



Another way to describe lines on plates

Suppose ye plate bee abc, whose edg boc is to be made into ye fashion of a given crooked line suppose (o) is its vertex & yt a circle described wth ye Radius εo would bee as crooked as ye given line at its vertex. Againe suppose two straight rulers mn & pq to bee very trew & steddyly fastened together wch must a very little incline ye one to ye other, soe as that being produced they would meete at r: Then are ye lines $pn=a$ [illegible], & $pr=b$ given.



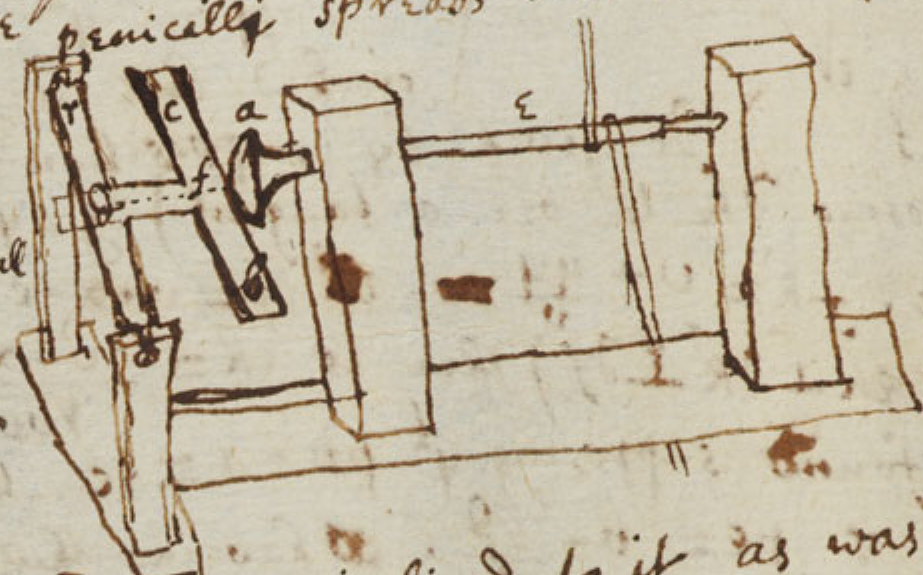
Suppose yn ye point d in ye crooked line is to bee found yn is dc given by supposition, & consequently (supposing dk to bee a tangent) $dg=y$. $gc=x$. $fg=v$. $fd=s$ $\varepsilon c=c$. $fk=v+\frac{yy}{v}$. $\varepsilon f=v+x-c$. $\varepsilon k=c-x+\frac{yy}{v}$. & (if $\varepsilon k \perp dk \perp df$) then is $\varepsilon k=\frac{cv-xv+yy}{\sqrt{vv+yy}}=d$. ($\varepsilon k$) being thus found, supposing yt $pn=a=\varepsilon c$, then I take $r\varepsilon=\frac{bd}{a}$. that is $p\varepsilon=b-\frac{bd}{a}$. having thus found ye point ε lay ye plate twixt ~~[illegible]~~ the two rulers so yt ye point of it, fall upon ye point ε yn should ye line mn touch ye plate in d. But note yt $pn \perp mn$.

In both telescopes & microscopes tis most convenient to have a convex glasse next ye eye for by that meanes ye angle of vision will bee much greater yn it will bee wth a concave one (though both doe magnifie alike). If ye convex glasse be Hyperbolicall (&c) make it soe bigg yt ye penecilli may crosse in ye pupill; yt is, ye exterior focus will be as far distant from ye vertex as ye eye is: let ye glasse bee as thinn [illegible] as may bee yt ye eye bee not too farr from ye vertex yet it should bee about as thick as ye distance of ye interior focus from ye vertex.

And by this meanes also, (ye focus of ye object=glasse being within ye telescope twixt ye glasses) there may bee placed at that focus ye edge of

a steele ruler accurately divided into equall parts (to measure ye diameters or distances of starrs &c) wch should bee soe made yt by a pinne or handle it may be placed in any posture & in any parte of ye focus, wth out otherwise altering ye Telescope in ob= servations.

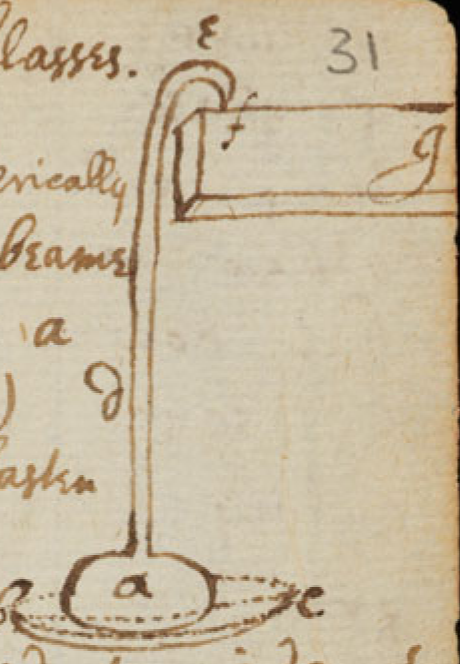
Note that were not ye glasses faulty they would not onely magnify objects but render vision more distinct; each of the penicilli passing through (perhaps but) the 10th, 20th or 100th parte of ye pupill must bee more exactly refracted to one point of ye Tunica Retina yn in ordinary visiō in wch each of ye penicilli spreads over all the pupill.



☞ Note also that yt ye glasse a may be ground ~~Parabolar~~ Hyperbolical by ye line cb, if it turne on ye Mandrill ε whilst cb turnes on ye axis rd being inclined to it as was shewed before. If the Edge (cb) bee not durable enough, inough instead thereof use a long small cilinder. wch I conceive to bee the best way, of all. for a Cilinder of all sollids is most easily made exact (being turned, as in the figure, by a gage untill its thicknesse bee every where equall).

2 the Cilinder may bee made ~~~~ to slip up & downe & turne round whereby it will not onely grinde ye glasse crosse wise to take of all rubbes, but also ye glasse & cilinder will grinde ye one ye other truer & truer. All ye difficulty is in placing ye axis rd perpendicular to the Mandrill aε, of vertex to vertex wch yet may bee done exactly severall ways. & untill yn the glasse & cilinder will not fit. & should ye axis not intersect ye glasse would bee still Hyperbolicall except a point at the vertex of it. The same instrument may also serve for severall glasses onely making If longer or shorter. Let the Cilinder hang over the glasse.

To Grinde Sphæricall optick Glasses.



If ye glasse (bc) is to bee ground sphærically hollow: Naile a steele plate to ye beame (fg), on ye upper side: In wch make a center hole for ye steele point (f) of ye shaft (def): to wch shaft fasten a plugg (a) of stone or leade or leather &c: (wth wch you intend to grinde ye glasse (bc)): wch shaft & plugg being swung to & fro upon ye center f will grind ye glasse bc sphærically hollow.

The manner whereby glasses may bee ground sphærically convex may appeare by ye annexed figure (being ye former way inverted). Also ye plugg (a), in ye first / second figure, is ground sphærically convex / concave.

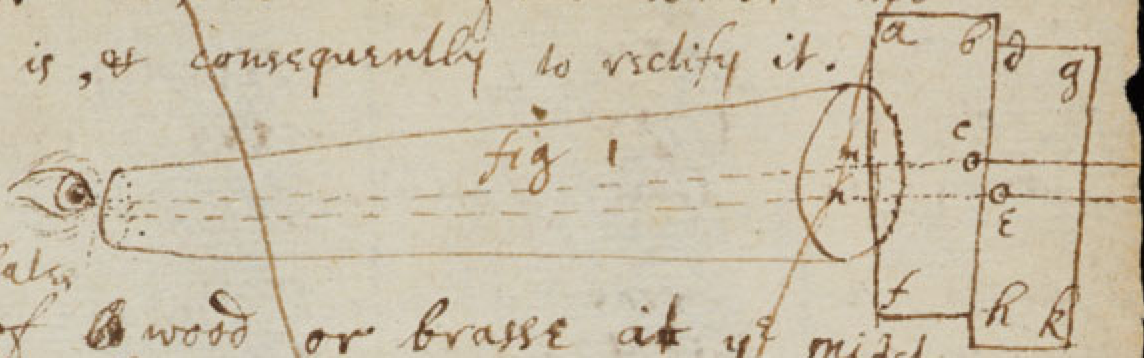
But if this way bee not exact enough yet hereby may bee grownd plates of mettall well nigh sphæricall, And by those plates may bee ground glasses after ye usuall manner. If a circular hoope of steele (abc) bee put about ye edge of ye glasse (d) to keepe it from grinding away at ye edges faster yn in ye middle.

But the best way of all will bee to turne ye glass circularly upon a mandrill whilest ye plate is steadily rubbed upon it or else

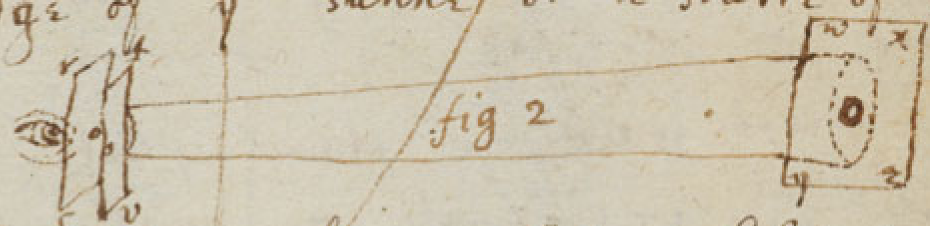
to turne ye plate upon a mandrill whilest
ye glasse is rubbed upon it (or let sometimes ye one, sometimes ye other bee turned.): & by this meanes
they will either of them weare the
other to a truely sphericall forme.
but however let there bee a hoope or
of some mettall wch weares more diffi-
cultly then glasse to defend ye glasse
from weareing more at its edges then
in ye middle. Perhaps it may doe well first to
weare ye plate sphæricall by ye hoope
alone wthout the glasse.
 The same meanes may bee used for grinding
plaine glasses.

Let not an object glasse bee ground
sphærically convex on both sides, but sphærically
convex on one side & ~~concave or plaine~~ plane or but a little convex on
ye other, & turne ye convexest side towards ye
object.

If the Glasses of a Telescope bee not truely ground, how to find where the fault is, & consequently to rectify it.



Take two plates abfh, dgkh of wood or brasse at ye midst of ye sides of wch boare two very small holes ce (viz whose diameters are about ye 20th or 30th pte of an inch, that they may transmit but soe much light as may serve to see ye edge of ye sunne or a starre of ye first magnitude)

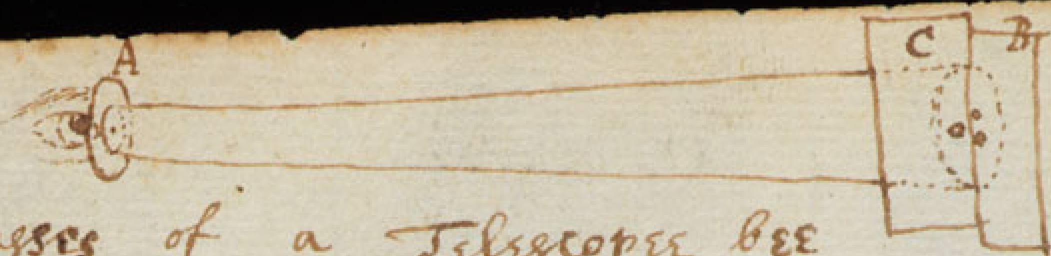


Also make two other plates rs, & tv like ye former but ye holes in ym must be as small as can bee (viz about ye 100th pte of an inch in diameter or lesse). ffor ye small end of ye tube.

Also make another plate wzxy, wth a hole in ye midst of it about ye 5t or sixt pte of an inch in diameter (viz equall to ye diameter of ye pupill of ye eye, or eye glasse). Make ye plate A like ye (but wth a very small hole) for ye eyeglass.

ffirst cover ye object glasse wth ye plates af, & gh ~~distant about ye~~ ye holes in ym being distant about ye sixt parte of an inch, & placed neare ye center of ye object glasse. Also cover ye eye glasse wth ye plate A soe yt its hole exactly respect ye center of ye eye glasse then turne ye tube to a starre wch will appeare like two starrs if ye tube bee too long or short, wch bee shortned or lengthned untill there appeare but one, And then is ye Tube of a good ~~distance~~ length for ye vertices of ye Glasses.

Secondly remove those plates, and instead thereof cover ye object glasse wth ye plate wz, its hole exactly respecting ye center of ye glasse. (ffig 2d)



If ye Glasses of a Telescope bee not truely ground. Theire errors ~~may bee~~ may bee thus found.

Because an error is much more easily discerna=ble in ye object glasse yn in ye eye glasse let us first suppose ye eye glasse to bee ground true towards its center, (tis exact enough if it be sphericall, & not Hyperbolicall), & so wee may find & rectifie ye errors of ye object glasse.

ffirst make a thin plate (A) of brasse & in the center of it a small hole (whose diame=ter perhaps may bee about ye 50th or 100dth parte of an inch. With wch plate cover ye eye glass ye center of it respecting ye center of ye glasse.

Secondly make two other plates the one B wth two holes as neare to its edge as may bee their distance being about ye 5t pte of an inch or lesse, & ye other C wth one hole close to ye midst of its edge. Let ye diameters of these 3 holes bee about a 20th pte of an inch or lesse. And theire edges must bee true that they may slide one upon another, & yet not let ye suns rays passe through, to wch purpose make ym oblique. Wth these two plates cover ye holes of ye other plate respecting the center of ye glasse) object glass (first stopping ye hole of C) & looke at a starre (or ye edge of ye sunne &c) & if ye object appeare double (like two starrs &c) make ye Tube longer or shorter untill it ap=peare single. Then open ye hole of C, & ye plate B being fixed, slide ye plate C up & downe still looking at ye starre, When ther appeares

but one starre y^t^ part of y^e^ glasse under y^e^ hole of C is truely ground in respect of y^e^ 2 parts of y^e^ glasse under y^e^ two holes of B. But [33] when y^e^ starre appeares double. And y^e^ po=
=sition of y^e^ starre caused by y^e^ hole of C in respect of y^e^ starre caused by y^e^ holes of B, shews ~~y^e^ error of y^e^ inclin~~ w^ch^ way y^e^ glasse under y^e^ hole of C is erroneously in=
=clined; the distance of y^e^ two starres giving y^e^ quantity of y^t^ ~~inclinato~~ error.

Thus y^e^ errors of y^e^ ~~of~~ object glasse bein found in every place of it they may bee all rectified, & found againe, & againe rectified, untill they almost or altogether vanish.

Then may y^e^ eye=glasse bee rectified much after y^e^ same manner, in every parte of it, & if it bee necessary y^e^ object glasse may bee againe rectified & againe y^e^ eye=glasse untill y^e^ telescope bee as perfect as y^e^ work=
=man can make. Whome perhaps experience ~~& other~~ may teach by this & y^e^ former rules to make telescopes as perfect as ~~may~~ men can hope to make them.

These Glasses may also bee rectified whilst on y^e^ Man=
=drill by observing y^e^ Images made by reflection from y^e^ vertex & all other pts of the glasse w^t^ proportion they have one to another & how much they are longer y^n^ broader in one place than another. &c.

The sines measuring refractions are in	Aire 42	water 56	Glasse 65	Christall 70
The proportions of the motions of the extreamely heterogeneous rays are in	39,4 . 40,4	$70\frac{3}{8} . 71\frac{3}{8}$	$95\frac{1}{10} . 96\frac{1}{10}$	$110\frac{1}{3} . 111\frac{1}{3}$
The proportions of ye sines of refraction of ~~the~~ extreamely heterogeneous rays into aire out of ~~out of aire~~	c	$90\frac{2}{3} . 91\frac{2}{3}$	68 . 69	$61\frac{4}{5} . 62\frac{4}{5}$
Their common sine of incidence	—	$68\frac{1}{[illegible]}$	$44\frac{1}{4}$	$36\frac{4}{5}$
Which substracted the difference is	—	$22\frac{1}{3} . 23\frac{1}{3}$	$23\frac{3}{4} . 24\frac{3}{4}$	24 . 25
The like proportions for refractions made into ~~out of~~ water out of			$275\frac{4}{5} . 276\frac{4}{5}$	$196\frac{1}{3} . 197\frac{1}{3}$
			$238\frac{2}{5}$	$157\frac{4}{9}$
			$37\frac{2}{5} . 38\frac{2}{5}$	$39\frac{1}{9} . 40\frac{1}{9}$

33b

34

33c

Contains various
Math. Investigations
relating to Mathematic
Areas of Curves, Music &c.
Also a memorandum of
date of discovery of the method
of infinite series signed
Is Newton

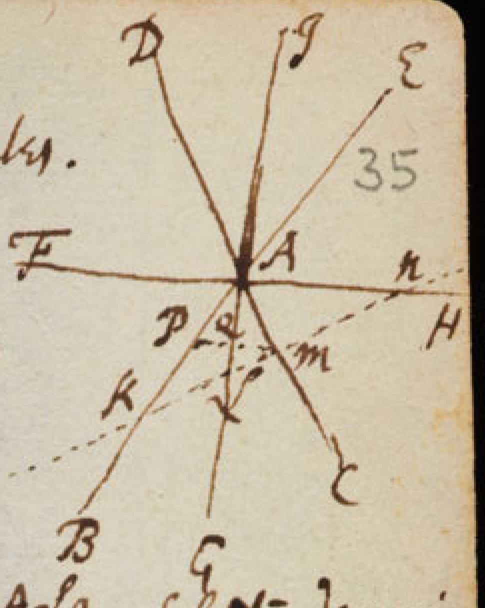
The sines measuring refractions are	Aire 42	water 56	Glass 65	Christall 70
The proportions of the motions of the extreamely heterogeneous rays are as	$39,4 . 40,4$	$70\frac{3}{8} . 71\frac{3}{8}$	$95\frac{1}{10} . 96\frac{1}{10}$	$110\frac{1}{3} . 111\frac{1}{3}$

The propo[illegible] refraction[illegible] generous [illegible] Their co[illegible] Which s[illegible] ... $62\frac{4}{5}$ ... $\frac{4}{5}$

The like [illegible] tions ma[illegible] ... 25 ... $197\frac{1}{3}$ ... $\frac{4}{9}$ ... $40\frac{1}{9}$

Theoremata varia.

Circa angulorum æqualitates.

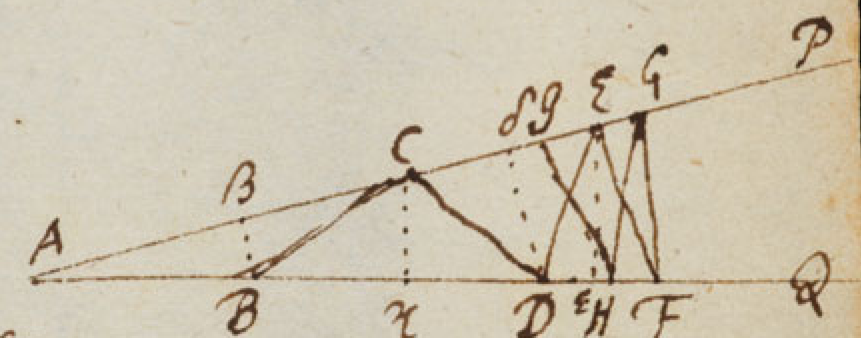


si ang DAB & DAE bisecentur a rectis FH et gG et ducatur quævis KLmn. Erit

1. AK.AM:: KL.LM::$_{\wedge}$ Kn.mn Euclid 6 3

2. AK x ~~AM~~ = AL q + KL x LM = ~~A~~ Kn x mn − AL q. Schoote de concin æquat

3. Am + AK. PK:: AQ.AL posito AP = Am.



Si in angulo quovis PAQ inscribantur æquales AB, BC, CD, DE, EF, FG, GH &c anguli ~~BAP~~ erit angulus CDQ duplus, ~~EDE~~ ECP tripl, EDQ quadr, FEP quint, GFQ sext, HGP sept. IHQ oct &c. Horu[m] vero angulorum posito radio AB sinus erunt Bβ, Cγ &c Cosinus Aβ, Bγ, Cδ &c. Ergo si $AB = r$ & $A\beta = x$ erit $_{\wedge}$ $A\gamma = \frac{2xx}{r}$ (AC = 2x). ~~$\frac{2xx - rx}{}$~~. $AD = ({}^2A\gamma - AB) = \frac{4xx - rr}{r}$. $A\delta = \frac{4x^3 - rrx}{rr}$ &c

Brenner. Basiliensis.

M.D.S.E. _

~~36~~
37

39

42

47

49

51

54

55

58

~~60~~
61

63

64

65

68

71

06

76

IV

Newton

$vy : \frac{1}{4}vv :: yy : \frac{\frac{1}{4}vvyy}{vy} = \frac{yv}{4}$

1663, 1664

Reasonings conc^d. cleare
the squaring of crooked lines.
Marcell notes
Method of finding areas

79*

9f
x6
a+6
a²+
a3+
a4+
a5 9
a6 48

To find the summe of y^e squares &c: of y^e rootes of an Equation.

If $a,\ b,\ c,\ d,\ e,\ f$ &c be the rootes of y^e Equation $x^6 + px^5 + qx^4 + rx^3 + sxx + tx + v = 0$. y^t is

$a+b+c+d+e+f = p\ (= g)$

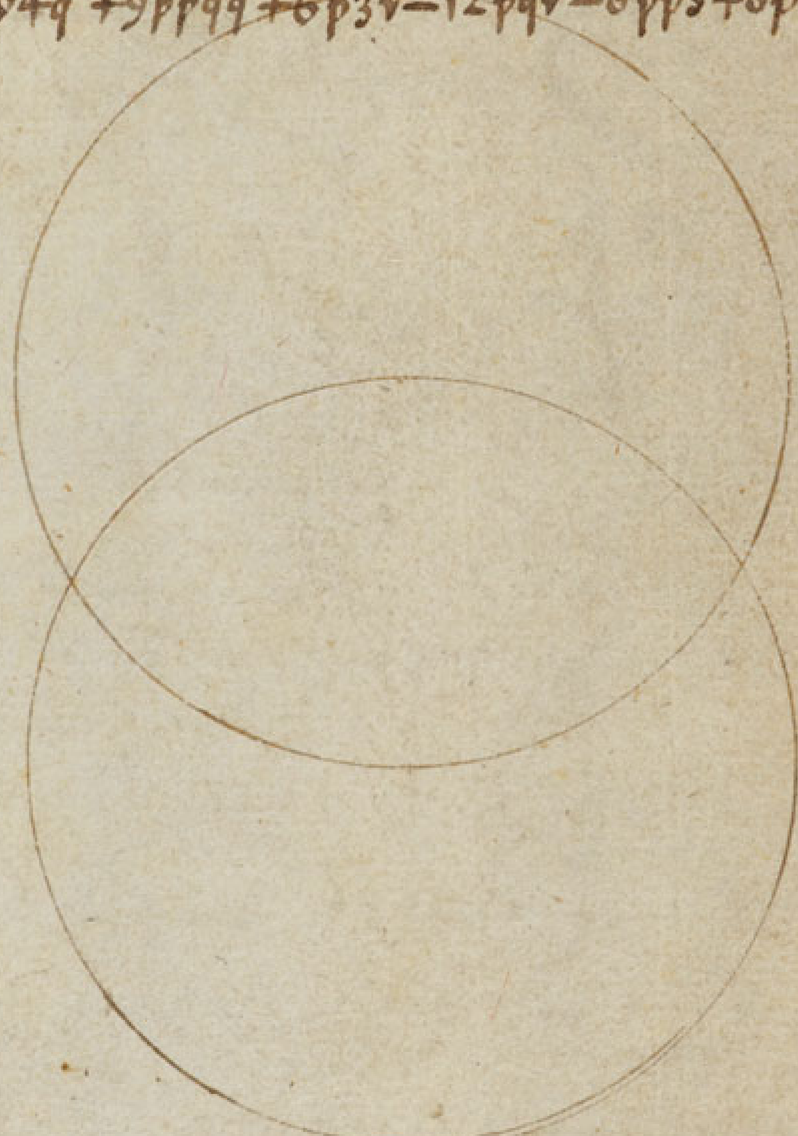
$a^2+b^2+c^2+d^2+e^2+f^2 = pp-2q.\ (= pg-2q = h)$

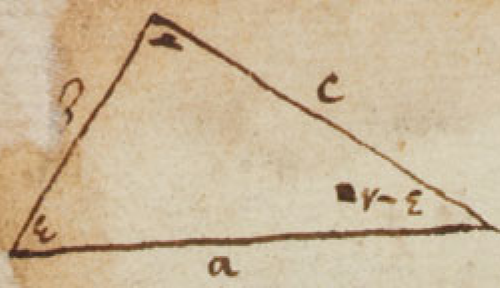
$a^3+b^3+c^3+d^3+e^3+f^3 = p^3-3pq+3r.\ (= ph-qg+3r = k)$

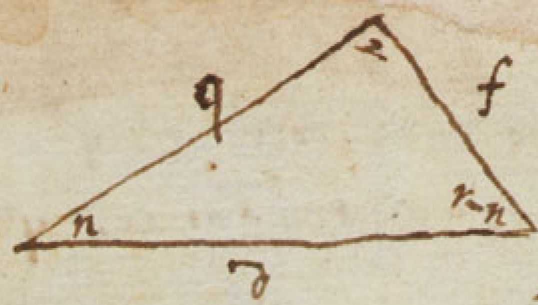
a^4+b^4 &c $= p^4-4ppq+4pr+2qq-4s.\ (= pk-qh+rg-4s = l)$

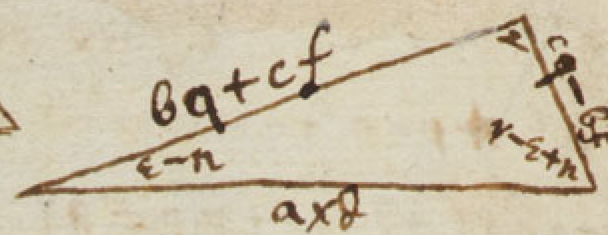
a^5 &c $= p^5-5p^3q+5pqq+5ppr-5ps-5qr+5t.\ (= pl-qk+rh-sg+5t = m$

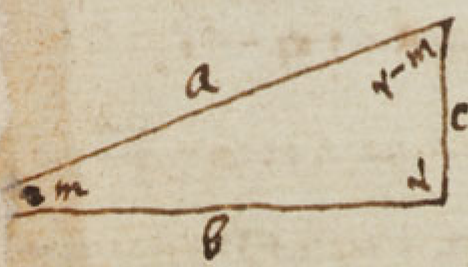
a^6 &c: $= p^6-6p^4q+9ppqq+6p^3r-12pqr-6pps+6pt-2q^3+3rr+6qs-6v.$

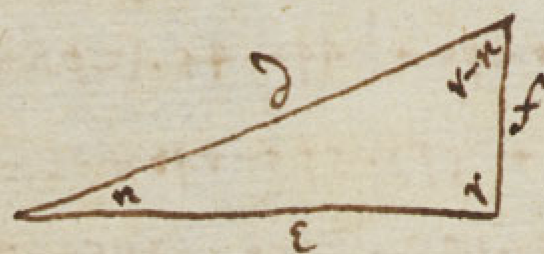



c
a
r−ε


q
f
n
r−n


bq+cf
ε−n
a×∂


a
c
r−m
m


f
n
ε


a∂
m+n
bε − cf

$ag = a$. $ab = x$. $bk = \frac{\delta x}{\epsilon}$. $be = y$. $bg = \sqrt{xx - aa}$

$gk = \sqrt{\frac{\delta\delta xx - \epsilon\epsilon xx + \epsilon\epsilon aa}{\epsilon\epsilon}}$. $\delta g = b$.

$c\epsilon = \frac{y}{\delta x}\sqrt{\delta\delta xx - \epsilon\epsilon xx + \epsilon\epsilon aa} = fg$.

$cf = \frac{\delta x + \epsilon y}{\delta x}\sqrt{xx - aa}$.

$\delta f = b - \frac{y}{\delta x}\sqrt{\delta\delta xx - \epsilon\epsilon xx + \epsilon\epsilon aa}$. $z^2 = \delta c^2 = \begin{cases} xx - aa + \frac{2\epsilon yx}{\delta} - \frac{2\epsilon yaa}{\delta x} + \cancel{\frac{2\epsilon\epsilon yy}{\delta\delta}} \\ + bb + yy. \\ - \frac{2by}{\delta x}\sqrt{\delta\delta xx - \epsilon\epsilon xx + \epsilon\epsilon aa} \end{cases}$

$\frac{2by}{\delta x}\sqrt{\delta\delta xx - \epsilon\epsilon xx + \epsilon\epsilon aa} = xx + yy - zz + bb - aa + \frac{2\epsilon yxx - 2\epsilon yaa}{\delta x}$.

$\cancel{bb\delta\delta xx}$

$$\begin{array}{llllll} \delta\delta x^6 + 4\delta\epsilon yx^5 & +2\delta\delta yyx^4 & -4\delta\epsilon aayx^3 & -4bb\delta\delta yyxx & -4\delta\epsilon y^3aax & +4\epsilon^2a^4y^2 \\ & -2\delta\delta zzx^4 & +4\delta\epsilon y^3 & +4bb\epsilon\epsilon yy & +4\delta\epsilon yz^2a^2 & \\ & +2\delta\delta bb & -4\delta\epsilon yzz & -8aa\epsilon\epsilon yy & -4\delta\epsilon ybba^2 & \\ & -2\delta\delta aa & +4\delta\epsilon ybb & +4^2\delta\delta y^4 & +4\delta\epsilon ya^4 & \\ & +4\epsilon\epsilon yy & -4\delta\epsilon yaa & & & -4bb\epsilon^2a^2y^2 \end{array}$$

Ad constructionem Canonis angularis.

$\frac{90^{gr}}{5} = 18^{gr}$. $\frac{18^{gr}}{5} = 3^{gr} + 36'$. Et $\frac{60^{gr}}{3} = 20^{gr}$. $\frac{20^{gr}}{3} = 6^{gr} + 40'$

$\frac{6^{gr} + 40'}{2} = 3^{gr} + 20'$. $3^{gr} + 36' - 3^{gr} - 20' = 16'$. $\frac{16'}{2} = 8'$. $\frac{8'}{2} = 4'$. $\frac{4'}{2} = 2'$

$\frac{2'}{2} = 1'$.

If r = radius. Then

y^e sine of

78^{degr} is, $\frac{r\sqrt{5} - r + r\sqrt{30 + 6\sqrt{5}}}{8}$.

66^{degr} is, $\frac{r\sqrt{5} + r + r\sqrt{30 - 6\sqrt{5}}}{8}$.

42^{degr} is, $\frac{-\sqrt{5rr} + r + \sqrt{30rr + 6rr\sqrt{5}}}{8}$.

6^{degr} is, $\frac{\sqrt{30rr - 6rr\sqrt{5}} - \sqrt{5rr} - r}{8}$.

Suppose $gh = x$. $nh = r$. Then

$gh = x$. unisectio

$ab \times r = 2rr - xx$. bisectio

$\overset{2}{h}b \times r^2 = 3rrx - x^3$. trisectio

$\overset{3}{a}b \times r^3 = 2r^4 - 4rrxx + x^4$. quadrisec.

$\overset{4}{h}b \times r^4 = 5r^4x - 5rrx^3 + x^5$. quintusect.

$\overset{5}{a}b \times r^5 = 2r^6 - 9r^4x^2 + 6rrx^4 - x^6$.

$hb \times r^6 = 7r^6x - 14r^4x^3 + 7rrx^5 - x^7$.

$ab \times r^7 = 2r^8 - 16r^6x^2 + 20r^4x^4 - 8rrx^6 + x^8$.

$hb \times r^8 = 9r^8x - 30r^6x^3 + 27r^4x^5 - 9rrx^7 + x^9$.

$ab \times r^9 = 2r^{10} - 25r^8x^2 + 50r^6x^4 - 35r^4x^6 + 10rrx^8 - x^{10}$.

$hb \times r^{10} = 11r^{10}x - 55r^8x^3 + 77r^6x^5 - 44r^4x^7 + 11rrx^9 - x^{11}$.

As on y^e other leafe excepting some signes have changed.

If $gh = x$. $bh = y$. Then

$y = bh$. duplicatio anguli hag

$yy - xx = x \times h^2b$. triplicatio anguli hag.

$y^3 - 2xxy = xx \times h^3b$. quadruplicatio.

$y^4 - 3xxyy + x^4 = x^3 \times {}^4bh$. quint.

$y^5 - 4xxy^3 + 3x^4y = x^4 \times h^5b$. sext

$y^6 - 5xxy^4 + 6x^4yy - x^6 = x^5 \times hb$. sept

$y^7 - 6xxy^5 + 10x^4y^3 - 4x^6y = x^6 \times hb$. oct,

$y^8 - 7xxy^6 + 15x^4y^4 - 10x^6yy + x^8 = x^7 \times hb$. nonc

$y^9 - 8xxy^7 + 21x^4y^5 - 20x^6y^3 + 5x^8y = x^8 \times hb$. dec

$y^{10} - 9xxy^8 + 28x^4y^6 - 35x^6y^4 + 15x^8y^2 - x^{10} = x^9 \times hb$. und,

$y^{11} - 10xxy^9 + 36x^4y^7 - 56x^6y^5 + 35x^8y^3 - 6x^{10}y = x^{10} \times hb$. duod

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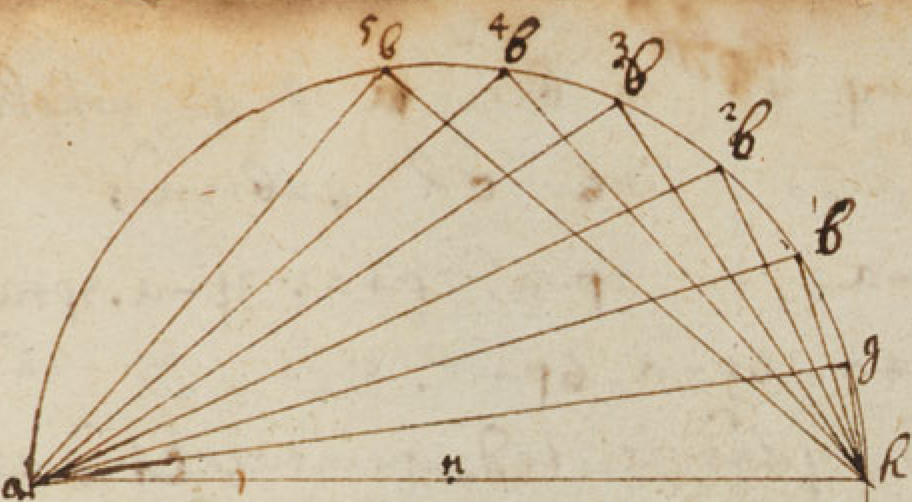


Of Angular sections

Suppose $ab=q$. $\frac{ak}{2}=r$. & $ag=x$.

& ye ye arches kg, gb, bb are equall.

By ye following Equations an angle bah may bee divided into any number of partes.

$x=q$ unisectio.

$x^2-2rr=rq$. bisectio.

$x^3-3rrx=rrq$. trisectio.

$x^4-4rrxx+2r^4=r^3q$. quadrisectio

$x^5-5rrx^3+5r^4x=r^4q$. quinquisectio

$x^6-6rrx^4+9r^4xx-2r^6=r^5q$. sexlusectio

$x^7-7rrx^5+14r^4x^3-7r^6x=r^6q$. septusectio.

$x^8-8rrx^6+20r^4x^4-16r^6x^2+2r^8=r^7q$.

$x^9-9rrx^7+27r^4x^5-30r^6x^3+9r^8x=r^8q$.

$x^{10}-10rrx^8+35r^4x^6-50r^6x^4+25r^8x^2-2r^{10}=r^9q$.

$x^{11}-11rrx^9+44r^4x^7-77r^6x^5+55r^8x^3-11r^{10}x=r^{10}q$.

$x^{12}-12rrx^{10}+54r^4x^8-112r^6x^6+105r^8x^4-36r^{10}x^2+2r^{12}=r^{11}q$.

$x^{13}-13rrx^{11}+65r^4x^9-156r^6x^7+182r^8x^5-91r^{10}x^3+13r^{12}x=r^{12}q$.

$x^{14}-14rrx^{12}+77r^4x^{10}-210r^6x^8+294r^8x^6-196r^{10}x^4+49r^{12}x^2-$ &c

$x^{15}-15rrx^{13}+90r^4x^{11}-275r^6x^9+450r^8x^7-318r^{10}x^5+140r^{12}x^3-$ &c

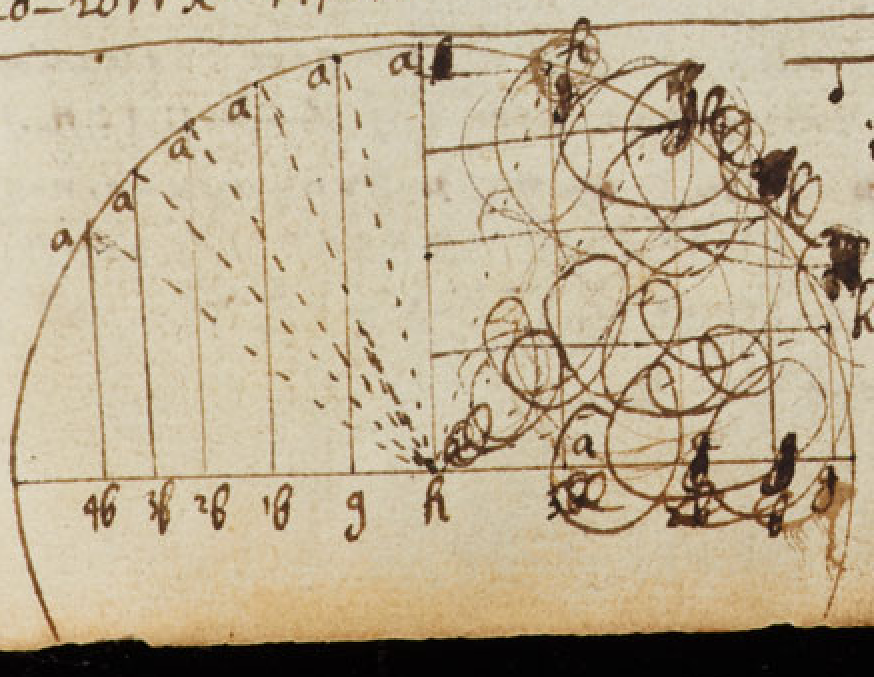
$x^{16}-16rrx^{14}+104r^4x^{12}-352r^6x^{10}+660r^8x^8-672r^{10}x^6+336r^{12}x^4-$ &c

$x^{17}-17rrx^{15}+119r^4x^{13}-442r^6x^{11}+935r^8x^9-1122r^{10}x^7+714r^{12}x^5-$ &c

$x^{18}-18rrx^{16}+135r^4x^{14}-546r^6x^{12}+1287r^8x^{10}-1782r^{10}x^8+1386r^{12}x^6-$ &c

$x^{19}-19rrx^{17}+152r^4x^{15}-665r^6x^{13}+1729r^8x^{11}-2717r^{10}x^9+2508r^{12}x^7-$ &c

$x^{20}-20rrx^{18}+170r^4x^{16}-800r^6x^{14}+2275r^8x^{12}-4604r^{10}x^{10}+4290r^{12}x^8-$ &c



This scheame is ye forme inversed.

versâ paginâ.

Suppose y^e perifery bgk to bee a & y^e whole perifery to bee p. The line bk subtends these arches. $a.\ p-a.\ p+a.\ 2p-a.\ 2p+a.\ 3p-a.\ 3p+a.\ 4p-a.\ 4p+a.\ 5p-a.\ 5p+a.\ 6p-a.\ 6p+a.$ &c: All w^ch are bisected, trisected, quadrisected, quintusected &c after same manner. As for example

The rootes of y^e Equation $kb \times rr = 3rrx - x^3$. are 3. The first whereof subtends y^e arches $\frac{a}{3}.\ \frac{3p-a}{3}.\ \frac{3p+a}{3}.\ \frac{6p-a}{3}.\ \frac{6p+a}{3}.\ \frac{9p-a}{3}.\ \frac{9p+a}{3}$ &c. The second subtends y^e arches $\frac{p-a}{3}.\ \frac{2p+a}{3}.\ \frac{4p-a}{3}.\ \frac{5p+a}{3}.\ \frac{7p-a}{3}.$ &c. The 3^d $\frac{p+a}{3}.\ \frac{2p-a}{3}.\ \frac{4p+a}{3}.\ \frac{5p-a}{3}\ \frac{7p+a}{3}$ &c.

Soe y^e rootes of y^e Equation $kb \times r^4 = 5r^4x - 5rrx^3 + x^5$, doe y^e first subtend y^e arches $\frac{a}{5}.\ \frac{5p-a}{5}.\ \frac{5p+a}{5}$ &c: y^e 2^d $\frac{p-a}{5}.\ \frac{4p+a}{5}.\ \frac{6p-a}{5}.$ y^e 3^d $\frac{p+a}{5}.\ \frac{4p-a}{5}.\ \frac{6p+a}{5}$ &c. y^e 4^th $\frac{2p-a}{5}.\ \frac{3p+a}{5}.$ &c y^e 5^t $\frac{2p+a}{5}.\ \frac{3p-a}{5}.\ \frac{7p+a}{5}$ &c.

Hence may appeare y^e reason of y^e number of rootes in these equations & y^t y^e points of y^e circumference to w^ch they are extended, are æquidistant. & by y^e lower scheme may bee knowne w^ch rootes are affirmative & w^ch negative,

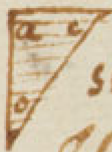
The numerall coefficients of y^e afforesaid Equations may bee deduced from this progression (if ∠ : ∠ :: 1 : n.)

$$1 \times \frac{0+n \times 1+n}{1 \times 1-n} \times \frac{n-2 \times n-3}{2 \times 2-n} \times \frac{n-4 \times n-5}{3 \times 3-n} \times \frac{n-6 \times n-7}{4 \times 4-n} \times \frac{n-8 \times n-9}{5 \times 5-n} \times \frac{n-10 \times n-11}{6 \times 6-n} \text{ \&c.}$$

As ~~if~~ $n=10$. y^e pression is $1 \times -10 \times \frac{-7}{2} \times \frac{-10}{7} \times \frac{-1}{2} \times \frac{2}{25} \times 0$. And y^e coefficients $1.\ -10.\ +35.\ -50.\ +25.\ -2.$

$1663\frac{3}{4}$ January.

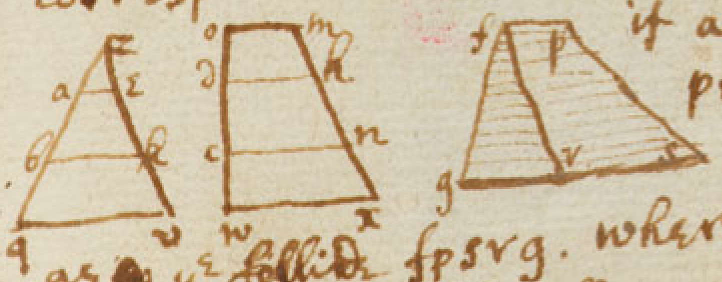
All y^e parallell lines w^{ch} can bee understoode to bee drawne uppon any superficies are equivalent to it, as all y^e lines drawne from (ao) to (co) may be used in stead of y^e superficies (aco.)



If all y^e parallell lines drawne uppon any superficies be multiplied by another line they produce a sollid like y^t w^{ch} results from y^e superficies drawne into y^e same line as if either all y^e lines in y^e superficies (oac) or if y^e superficies oac be drawne into y^e line (b) they both produce y^e same sollid (d) Whence

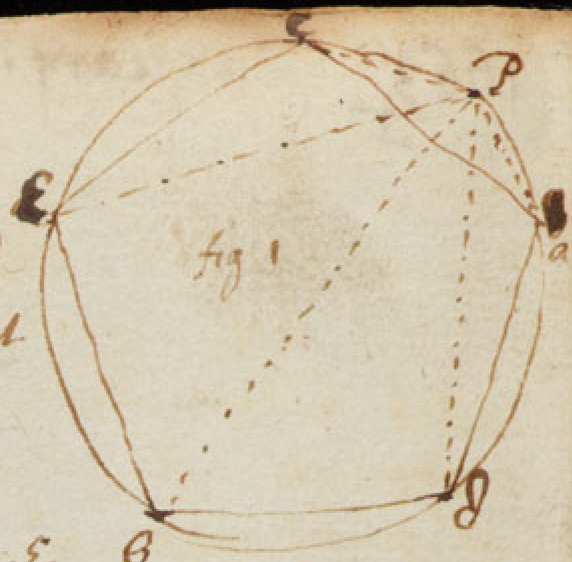
All y^e parallell superficies w^{ch} ~~ar~~ can bee understoode to bee in any sollid are equivalent to y^t sollid. And If all y^e lines in any triangle, w^{ch} are parallell to one of y^e sides, be squared there results a Pyramid. if those in a square, there results a cube. If those in a crookelined figure there results a sollid w^{th} 4 sides terminated & bended ~~after~~ according to y^e fasshion of y^e crookelined figure

If each line in one superficies bee drawne into each correspondent line in another superficies as in aebk & omnc if ae×dk. bk×cn. qv×wx &c. they produce a sollid whos opposite sides are fashioned by one of y^e superficies as y^e ~~sollide~~ fpsrg. where all y^e lines drawne from fr to ps are ~~equall to all~~ equall to all the correspondent lines drawne from ow to mx. & those drawne from fg to fr are equall to y^e correspondent lines drawne from qz to vz.



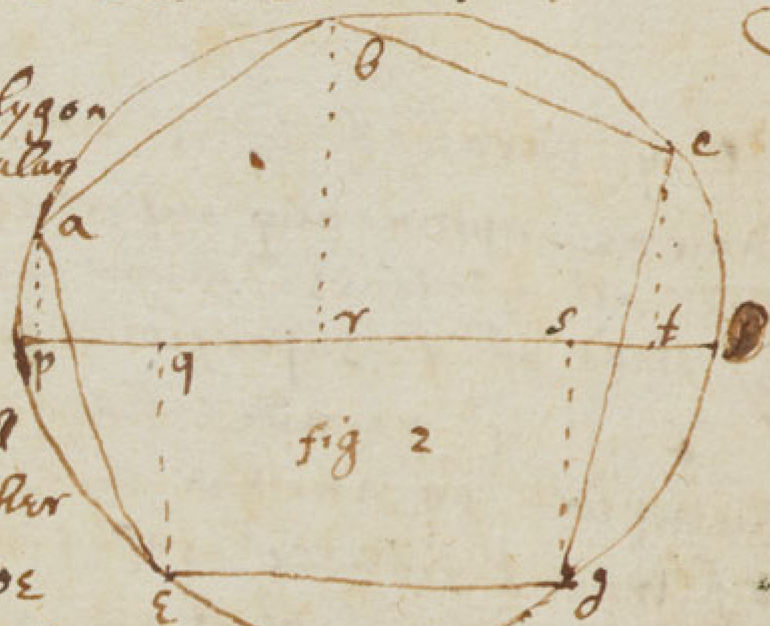
Theorema. 1

If in the Circle abcdeP there be inscribed any Poligon abcde w^th an odd number of sides, & from any point in y^e circumference P there bee drawne lines Pe, Pa, Pb, Pc, Pd to every corner of y^e Polygon: y^e summ of every other line ~~[illegible]~~ is equall to y^e summ of y^e rest, $Pa + Pb + Pc = Pd + Pe$. & soe are their Cubes $Pa^3 + Pb^3 + Pc^3 = Pd^3 + Pe^3$. unless y^e figure be a Trigon



Theor 2

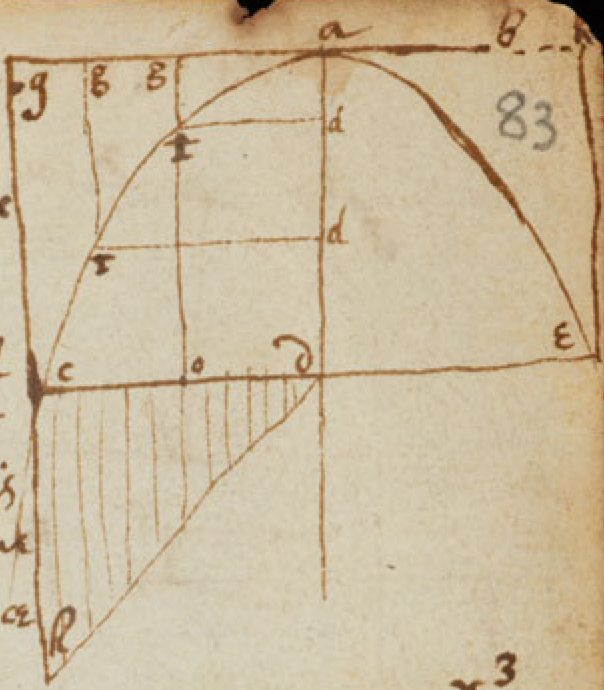
If from y^e points of y^e Polygon there bee drawne perpendiculars ap, br, ct, ds, eq to any Diameter pt: y^e summe of y^e Perpendiculars on one side y^e Diameter is equall to their summe on y^e other $ap + br + ct = eq + ds$. & soe is y^e summe of their cubes (unlesse w^n y^e figure is a Trigon), $ap^3 + br^3 + ct^3 = eq^3 + ds^3$. & of their square cubes (except w^n y^e figure is a Trigon or Pentagon. &c.



Theor 3

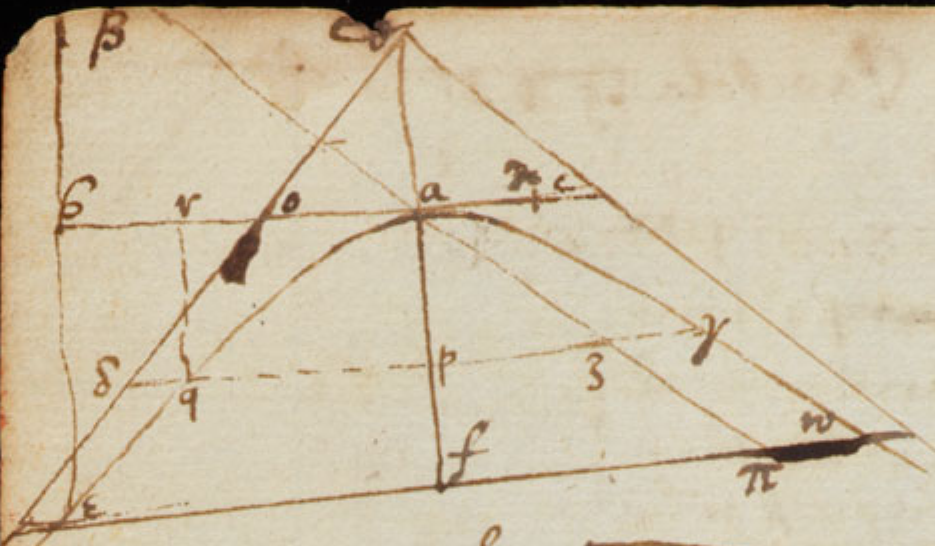
If y^e 2 Circles (fig 1 & 2) be equall w^th like Poligons inscribed, & Pa in fig 1 be assumed double to pa in fig 2. then are all y^e other correspon ding lines in fig 1 double to those in fig 2 viz $Pb = 2rb$, $Pc = 2tc$, $Pd = 2sd$, $Pe = 2qe$.

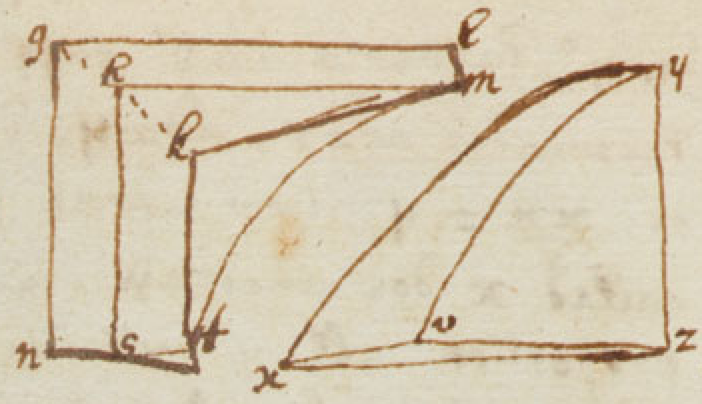
To square y^e Parabola

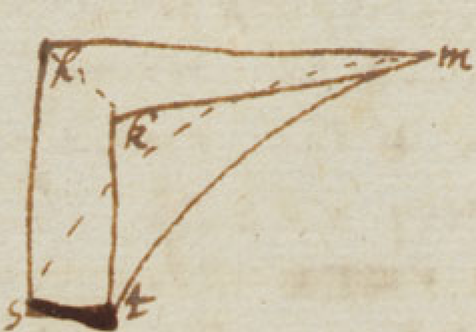


In y^e Parabola cae suppose y^e Parameter $ab=r$. $ad=y$. $dc=x$. & $ry=xx$ or $\frac{xx}{r}=y$. Now suppose ~~every~~ y^e lines called x does increase in arithmeticall proportion all y^e x's taken together ~~y^e supf~~ make y^e superficies dck w^ch is halfe a square let every line drawne from cd to kd be square & they produce a Pyramid equall to every $xx=\frac{x^3}{3}$. w^ch if divided by r there remaines ~~every~~ $\frac{x^3}{3r}=\frac{yx}{3}$ equall to every $\frac{xx}{r}$ equall to every (y) or all y^e lines drawne from ag to acec equall to y^e superficies agc equall to a 3^d p^te of y^e superficies adcg & y^e superficies $acd=\frac{2yx}{3}$.

Otherwise. suppose $ce=b$. $co=x$. $to=y$. & $ry=bx-xx$ y^e lines x increassing in arithmeticall proportion every x is equall to 4 times y^e superficies $cdk=\frac{bb}{2}$ w^ch drawne into b produce y^e sollid $\frac{b^3}{2}$ but if every x be squared they produce a pyramid equall to $\frac{b^3}{3}$. wherefore every $bx-xx=\frac{b^3}{6}$ equall to every ry equall to y^e superficies adce drawne into r & $\frac{b^3}{6r}=$ to cade as before.







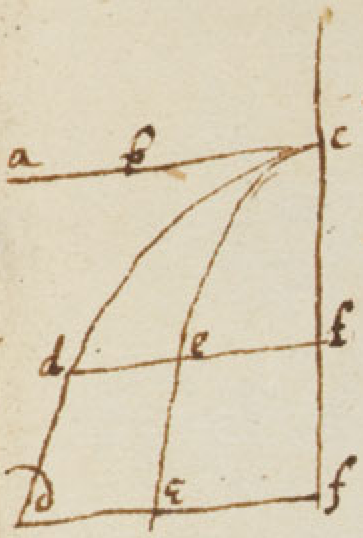
$*\, p\delta = p\vartheta = \frac{q+y}{5}$ & $\frac{qy+yy}{5} = xx$

To Square y^e Hyperbola

In y^e Hyperbola eqaw. suppose $ef=a$. $fa=b$. $ap=rq=y$. $pq=ar=x$. $ad=q=5oa=5ac=5b$. & $da+ax : ar :: ar : rq$.
$xx=by+qyx$. In w^{ch} equation every x taken together is equall to y^e triangle $a\beta b$ equall to $\frac{aa}{2}$ & every xx taken together is a pyramid $=\frac{a^3}{3}$.
Every y taken together is equall to y^e superficies $eba=mkt$. If y^n $gk=lm=ns=ax=b$. every by is equall to y^e solid $nglmks$. If y^e angle mkk is a right one & if $mk=gl==ba=ef=a$ ~~…~~ that is if y^e triangle $mkk=ab\beta$. every qx will be equall to y^e sollid $mkstk$. Joyne these two sollids together as in $lmtng=\frac{a^3}{3}$. *

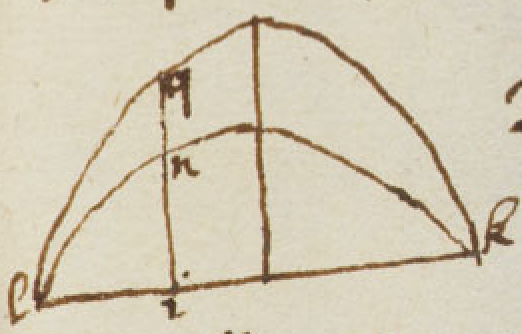
⊛ Againe suppose every x taken together to be equall to y^e superficies aef, & y^e line qy squared is $4xx$. every $4xx$ composeth a sollid like (π & δ) an eighth of w^{ch} (is equall to every $\frac{xx}{2}$) being like $xyzv$; xv will be equall to $\pi x=ef=a=st=xz=ox=km$. & $vz=\pi x=a\beta=km$. whence y^e convexe superficies xyv of y^e figure $xyvz$ will fitly joyne w^{th} y^e concave superficies mst of y^e figure $skmtk$. If every x is equall to y^e superficies aef, every y shall be equall to y^e triangle $af\pi=\frac{bb}{2}$. every $yy=\frac{bbb}{3}$. every $qy=\frac{qbb}{2}$ & therefor y^e sollid $yxzv=\frac{b^3}{30}+\frac{qbb}{20}=\frac{2b^3+3qbb}{60}$. Joyne y^e sollid $shmkt$ to $yxvz$ & there resulteth $shmztk = \frac{aab}{2}$ from w^{ch} againe substract $xvzy=\frac{2b^3+3qbb}{60}$ & there remaines y^e sollid $mkstk=\frac{30aab-2b^3-3qbb}{60}$ w^{ch} substract from y^e sollid $nt\,lmg=\frac{a^3}{3}$ & there remaines $nglmks=\frac{20a^3+2b^3+3qbb-30aab}{60}$ w^{ch} being divided by $b=\sqrt{\frac{5rr}{4}}$. there remaines $\frac{40a^3+4b^3+6qbb-60aab}{rr\sqrt{5}}$ = to y^e superficies abe

The squaring of severall croked lines of ye second kind.



In any two crooked lines I call ye Parameter or right side of ye greater (r). but of ye lesser (s). ye transverse side (q). ye right axis as cf (x) or ~~ef~~ = v. ye transverse axis as fe y, or fd z.

Suppose in ye Parab: ddc : ac = r. & in eec: bc = s. $rx = zz = df^2$. $sx = yy = fe^2$. $\sqrt{rx} - \sqrt{sx} = de = p$. $rx = sx + pp + 2p\sqrt{sx}$. $rx - sx - pp = 2p\sqrt{sx}$. Or $rrxx - 2rsxx + ssxx - 2pprx - 2ppsx + p^4 = 4ppsx$. Or $p^4 - 2rxpp - 6sxpp + rrxx - 2rsxx + ssxx = 0$. if $p = y$. $xx = \frac{+2ryy\,x + 6syy\,x - y^4}{rr - 2rs + ss}$. make $cf = a$. $fd = b$. $fe = c$. $ceeff = \frac{2ac}{3}$ & $cddeff = \frac{2ab}{3}$ therefore $\frac{2ab - 2ac}{3} = cddee$ ye square of ye crooked line cdd (when ye line cee is supposed to close wth ye line cf) whose nature is exprest by ye foregoeing Equation.



2 $lk = b$. $li = x$. $qi = y$. $in = z$. $\cancel{bb} + bx - xx = ry$. $\cancel{bb} + bx - xx = sz$. $\frac{\cancel{bb} + bx - xx}{r} = y$. $\frac{\cancel{bb} + bx - xx}{s} = z$.

$qn = \frac{-sxx\ \cancel{bbs} + bsx + \cancel{bb}r\ [illegible]\ brx}{rs} = v = y$

or, $r\,xx - \cancel{[illegible]}\ bx - rsy = 0$. ~~[illegible]~~

$xx - bx - \frac{rsy}{r-s} = 0$

3 $tg = x$. $dg = z$. $gp = y$. $rx - rz = dp^2$.

$rx - rz + zz = y^2$. $zz = rz - rx + yy$

~~$r : a :: \sqrt{xr - rz} : z$.~~

~~$z = \frac{r}{2} \pm \sqrt{\frac{1}{4}rr - rx + yy}$.~~

~~$\frac{1}{2}rr \pm r\sqrt{\frac{1}{4}rr - rx + yy} = a\sqrt{rx - rz}$.~~

~~$\frac{1}{4}r^4 + rryy - r^3x - aarx + rzaa$~~

~~$\frac{x}{2} = \pm\sqrt{\frac{1}{4}rr - rx + yy}$.~~

$\cancel{\frac{1}{4}v^6 + v^4 yy + vvy^4 \quad v^5x - 2v^3yyx + v^4xx}$
$\cancel{-v^3aax - 2vyyaax + 2vvaaxx}$
$\cancel{+a^4xx}$

$v : a :: vx - vz : zz.$ $vzz = avx - avz.$ $zz = -az + ax.$

$z = -\frac{1}{2}a + \sqrt{\frac{1}{4}aa + ax}$. $ax - az = vz - vx + yy$ Or

$-yy + vx + ax = -\frac{1}{2}aa - \frac{1}{2}av + \frac{a}{v}\sqrt{\frac{1}{4}aa + ax}.$

186 / 31   354 / 59 / 944

$y^4 \begin{matrix} -2vx \\ -2ax \\ -aa \\ -av \end{matrix} yy \begin{matrix} +vvxx \\ +2avxx \\ +aaxx \end{matrix} \begin{matrix} +\frac{1}{4}a^4 \\ +\frac{1}{2}a^3v \\ +\frac{1}{4}aavv \end{matrix} \begin{matrix} +2aavx \\ +avvx \\ +a^3x \end{matrix} = \frac{1}{4}a^4 + a^3x + \frac{1}{4}vvaa + vvax$

$y^4 \begin{matrix} -2vx \\ -2ax \\ -aa \\ -av \end{matrix} yy \begin{matrix} +vvxx \\ +2avxx \\ +aaxx \\ +\frac{1}{2}a^3v \\ +2aavx \end{matrix} = 0.$ $xx \begin{matrix} +2aavx \\ -2vyy \\ -2ayy \end{matrix} \begin{matrix} +y^4 \\ -aayy \\ -avyy \\ +\frac{1}{2}a^3v \end{matrix} = 0$

$vv + 2av + aa$

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4 In y^e ~~Hyperbola~~ Parabola

$cb=a$. $b\epsilon=x$. $2aa-2ax-aa+2ax-xx=\epsilon d^2$

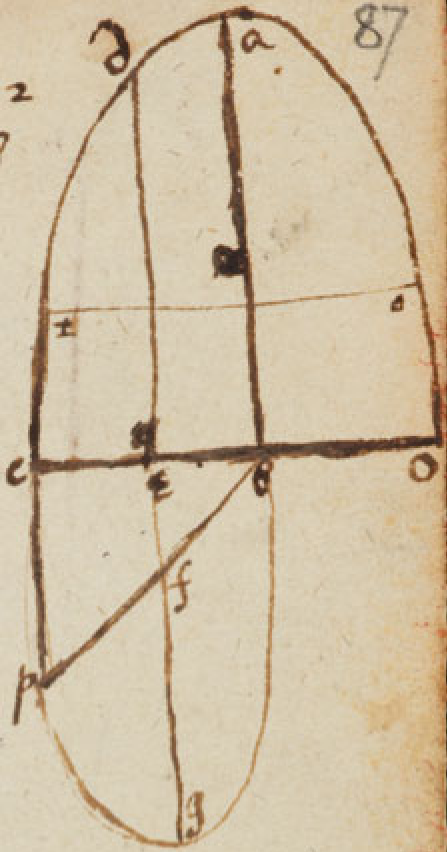
$aa-xx=\epsilon d^2=ry$. $\frac{aa-xx}{r}=y$. $cp=cb$

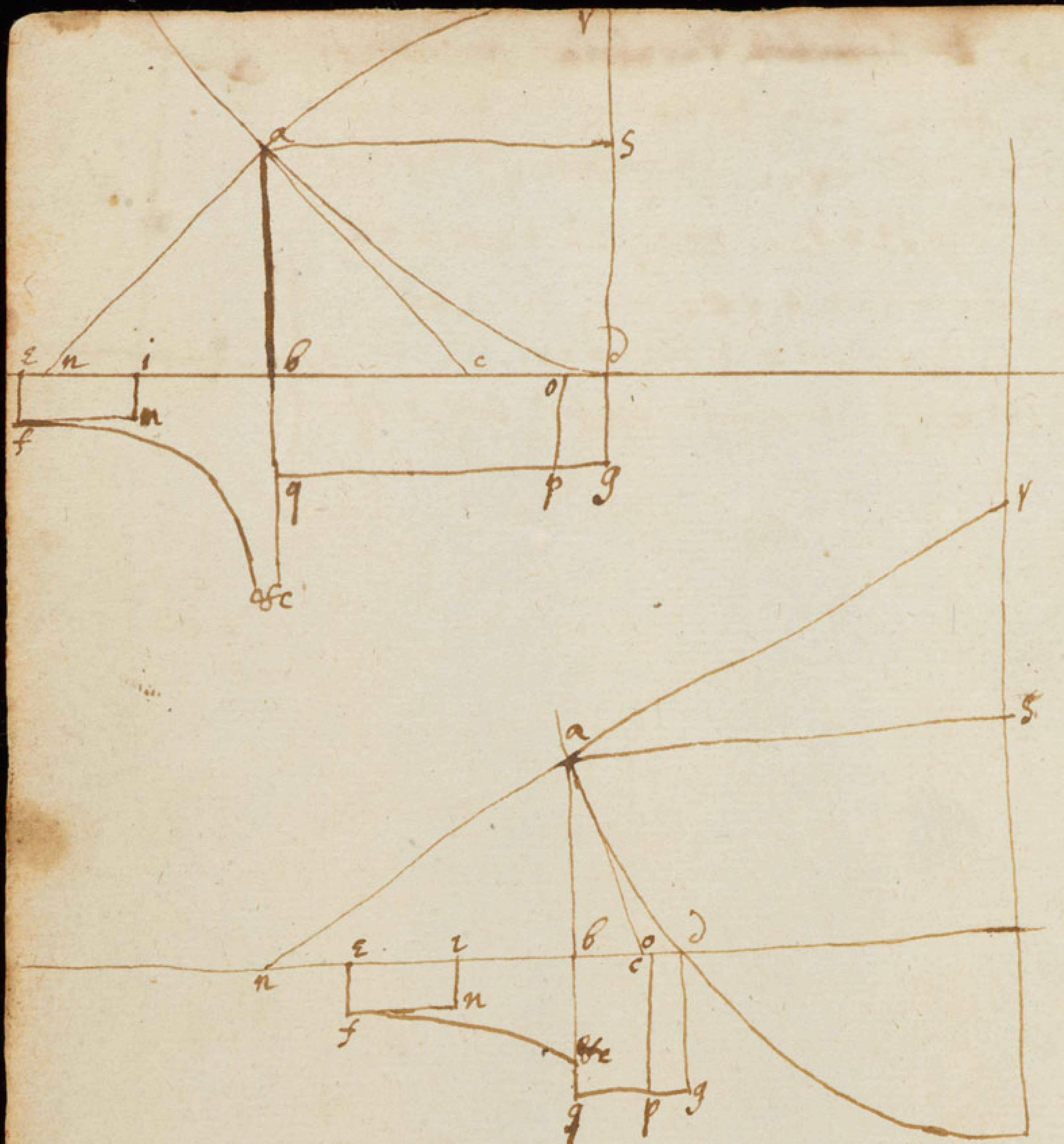
$r\epsilon b\times df=fg\stackrel{xc}{=}zc$. $\frac{aax-x^3}{r}+xx=zc$

$x^3-rxx-aax+rcz=0$. Since all

$\epsilon b\times df=\frac{1}{8}$ all $co^2=\frac{1}{4}ab\times ab\times r$. $ab=b$.

all $\epsilon b^2=\frac{a^3}{3}$ therefore $bgpf$ $\frac{1}{4}bbr+\frac{a^3}{3}$.

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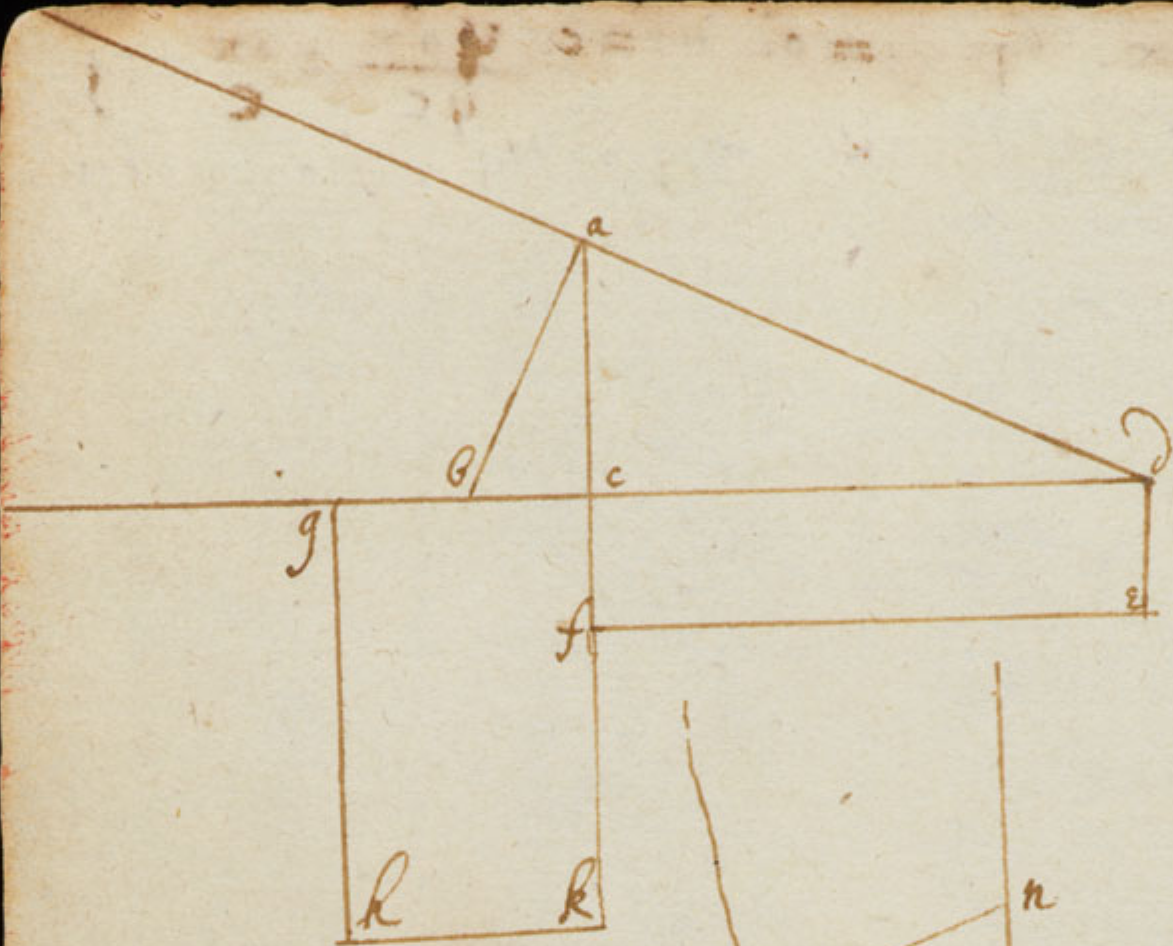
$ab = b\varepsilon = y$. $bd = x$. $bq = dg = b$. $rb = c$. $\frac{ybx}{yc} = \frac{bx}{c} = \varepsilon f$

Then shall bq &c: be y^e axis of gravity in $f\varepsilon b$ &c: & $bqgd$.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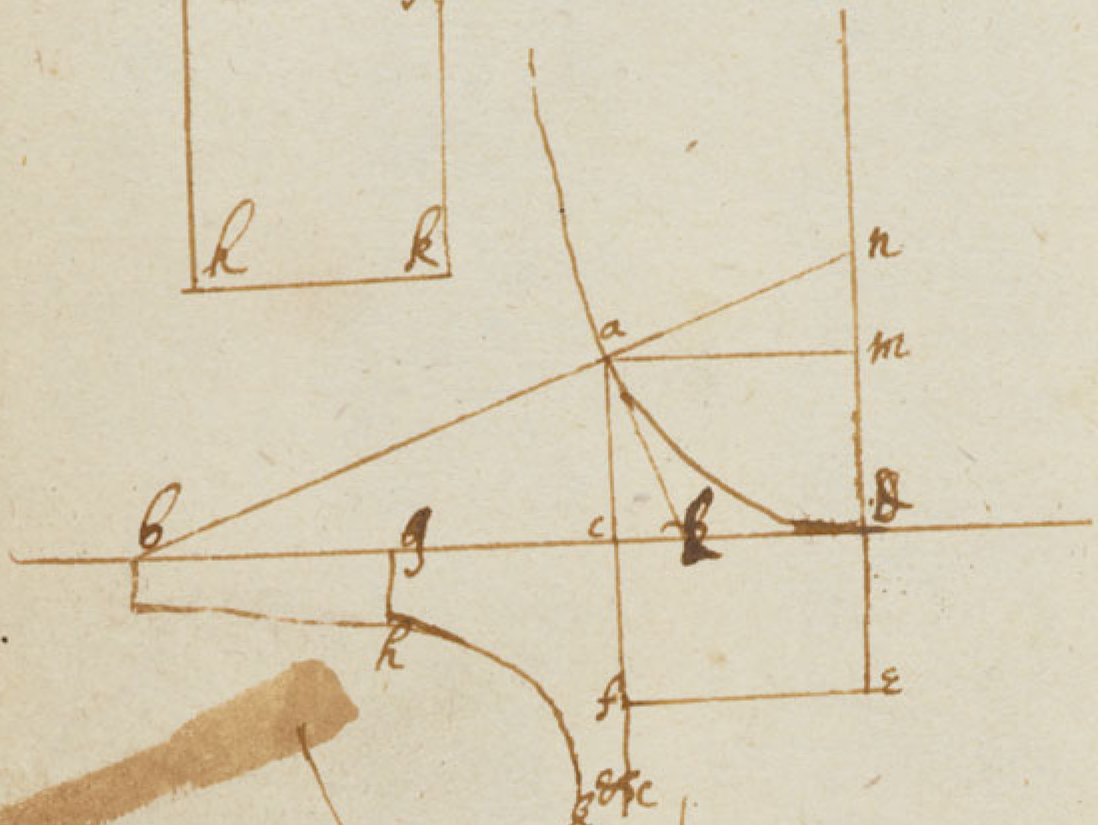
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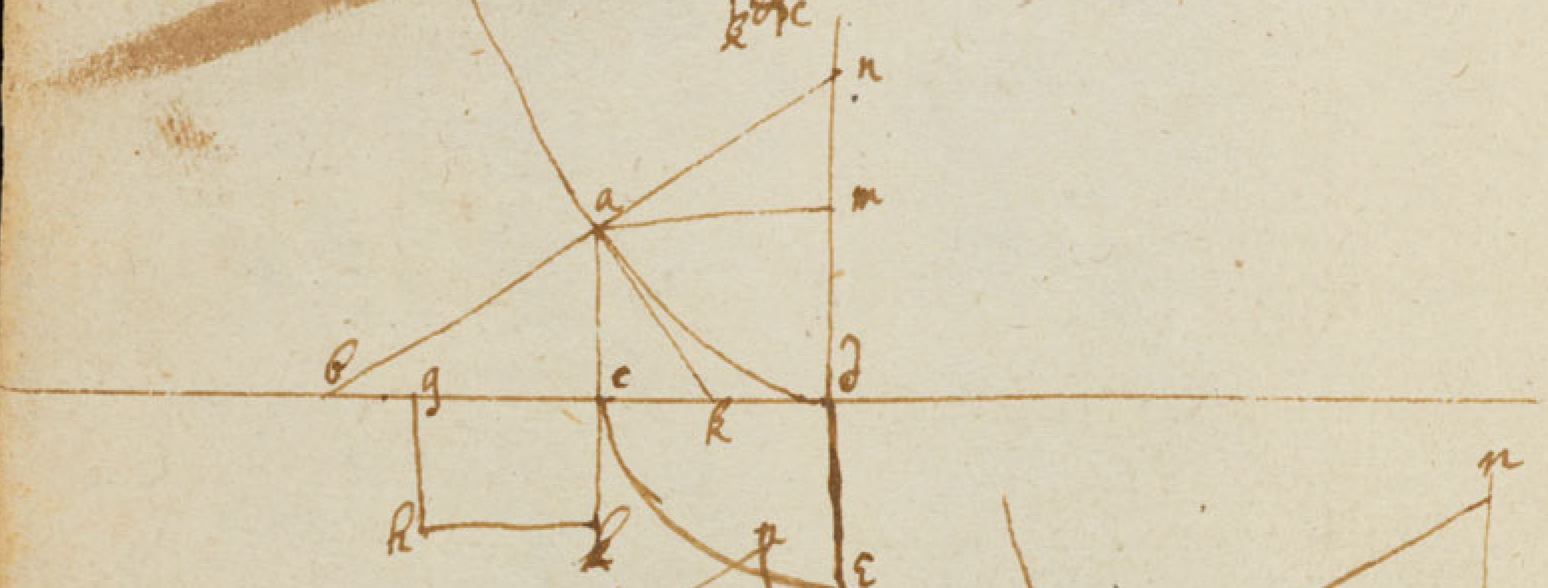
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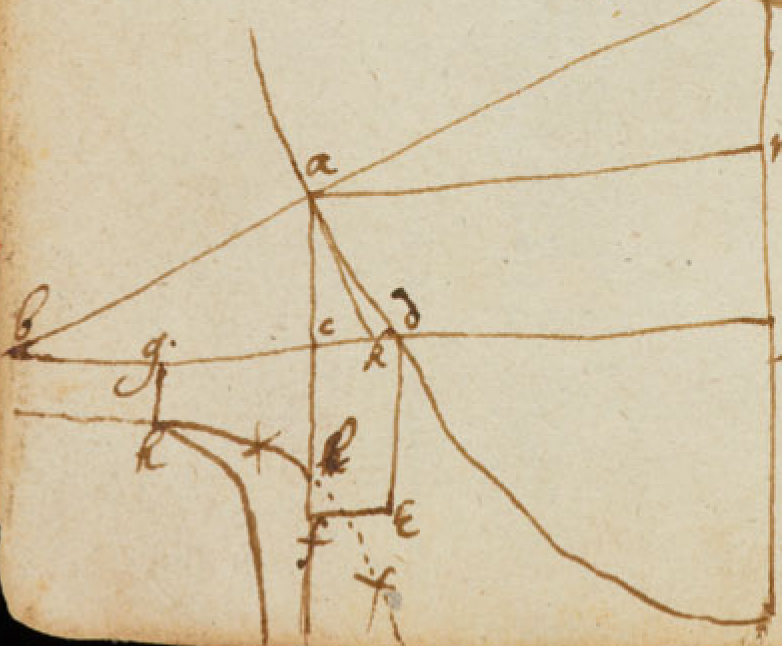
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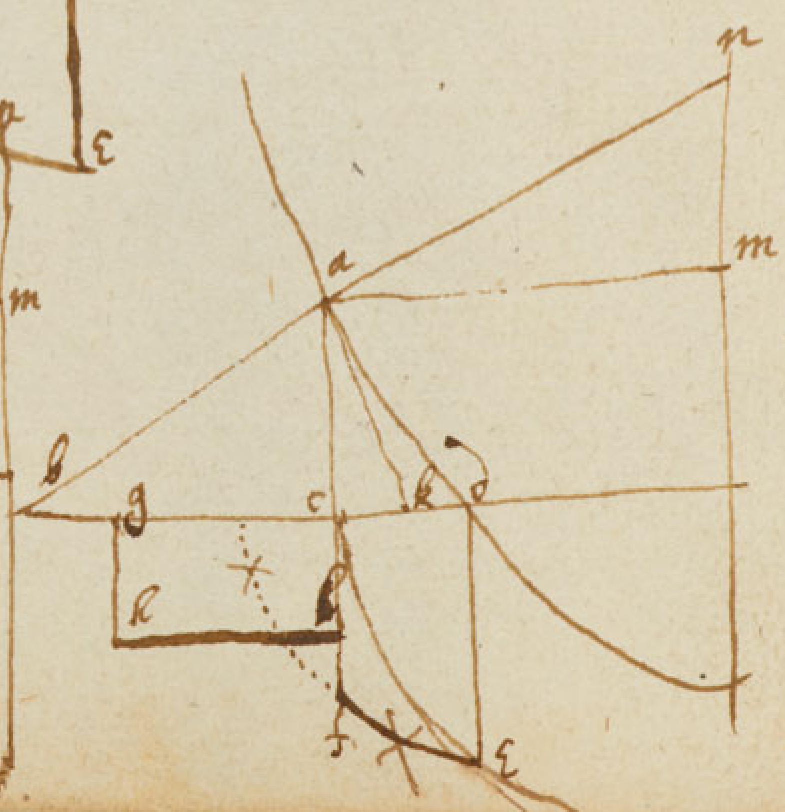
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In ye 1st figure.

$gc : cd :: cfed : ckhg = \frac{cd \times cfed}{gc}$. $ac = gc$.

$x : z :: za : xy$. $\frac{zza}{x} = ckhg$. or $\frac{xxy}{z} = cdef$. Suppose

$cd : ca :: ac : bc ::$ ye swiftnesse of de : to ye swiftnesse of gh.

$de \times$ its swiftnes : $gh \times$ its swiftness :: $gc : cd$.

$de \times cd : gh \times ca :: de \times ac : gh \times bc :: gc : cd$.

Fig 2d. 3d.

$ck : ca :: ac : bc :: nm : am ::$ swiftness de : swiftnesse gh.

$de \times$ its swiftnesse : $gh \times$ it swifnes $:: ck \times de : gh \times ac :: de \times ac : gh \times bc$

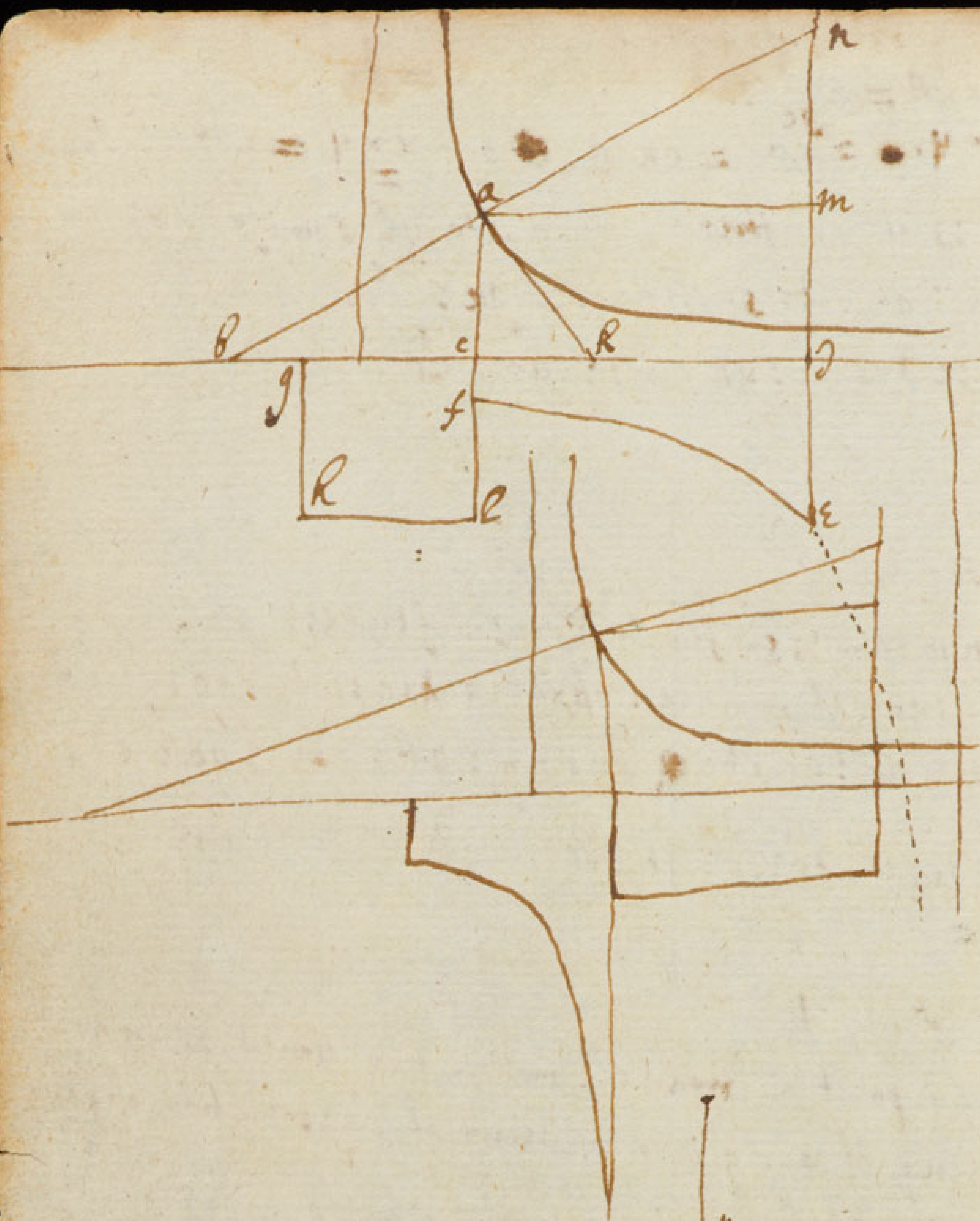
$de \times ck : gh \times ac :: de \times ac : gh \times bc :: de \times nm : gh \times am :: gc : cd$.

$de \times ck \times cd = gh \times ac \times gc$. $de \times cd = gh \times bc$. $de \times nm = gh \times gc$.

Fig 4

$ck : ca ::$ motion of ye point a from c : motion of ye point a from m :: $ck : ca ::$ increasing of $ac = gc$: increasing of cd :: motion of gh : motion of de. &c as before.

These are to find such figures $eghk$, $cfed$, as doe equiponderate in respect of ye axis $acfk$.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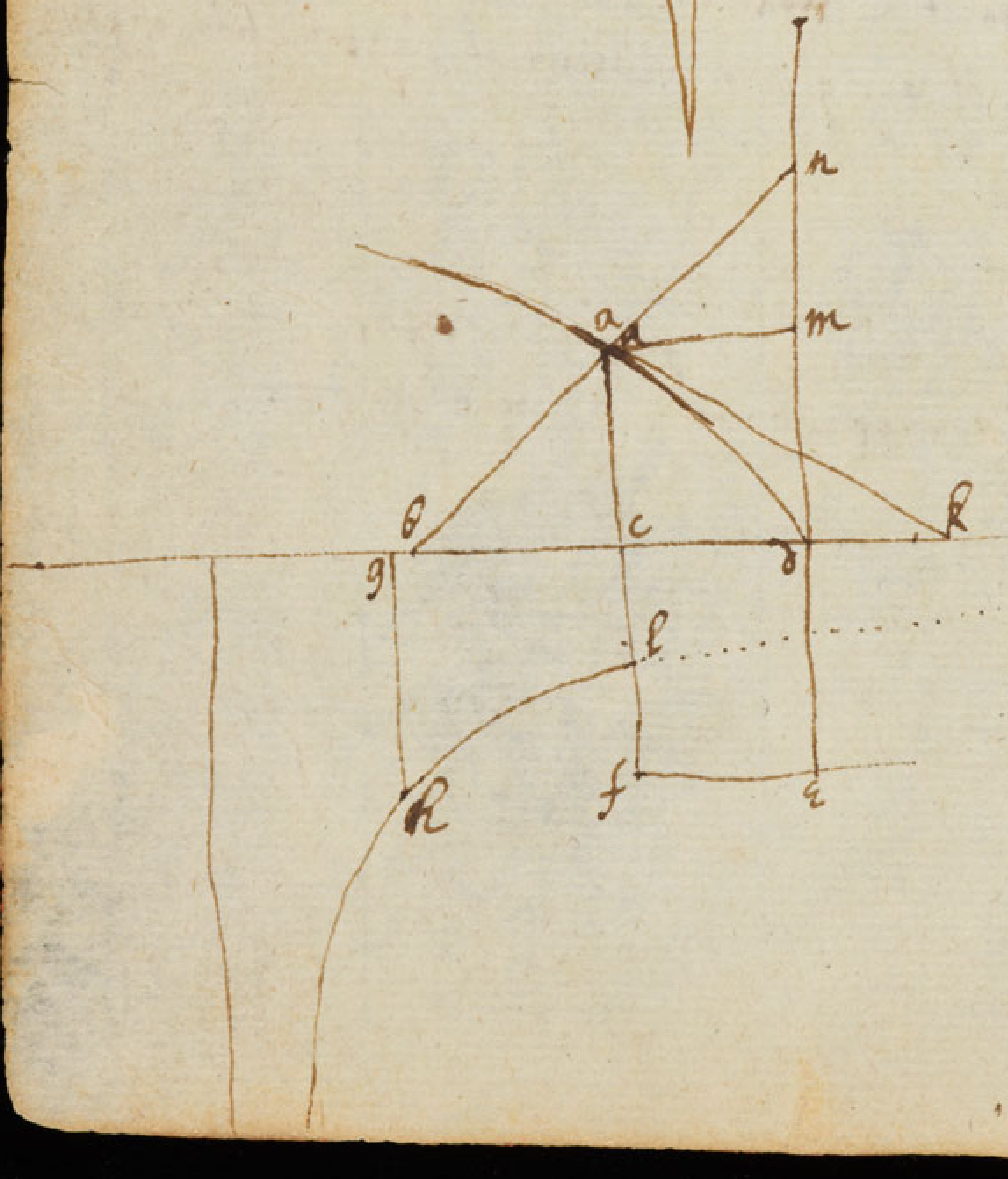
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Reasonings Concerning chance.

If ~~by one of ye equall chances a I gaine p, by ye chance~~ the equall chances a, b, c; &c are such yt one of ym must necessarily happen, & yt if one of ye chances a happen I gaine p thereby, or q by one of ye chanes b, or r by one of ye chances c. My chance or expectation is worth $\frac{pa+qb+rc}{a+b+c}$

1 If p is ye number of chances by one of wch I may gaine a, & q those by one of wch I may gaine b, & r those by one of which I may gaine c; soe yt those chances are all equall & one of them must necessarily happen: My hopes or chance is worth $\frac{pa+qb+rc}{p+q+r}=A$. The same is true if p, q, r signify any proportion of chances for a, b, c.

2 If I bargaine for more yn one chance (viz: yt after I have taken ye gaines by my first chance, from the stake $a+b+c$; I will venter another chance at ye remaining stake) my second lott is worth ~~$\frac{pa+qb+rc}{a+b+c\times p+q+r}$~~ $A\frac{-AA}{a+b+c}=A-\frac{AA}{a+b+c}=B$. My third lot is worth $A\frac{-AA-AB}{a+b+c}=C$. My ffourth lot is worth $A\frac{-AA-AB-AC}{a+b+c}=D$. My fift lot is worth $A\frac{-AA-AB-AC-AD}{a+b+c}=E$. My sixt lot is worth $A-A\times\frac{A+B+C+D+E}{a+b+c}$. &c

As if 6 men (1. 2. 3. 4. 5. 6.) cast a die soe yt he gaines a who throws a cise first: since there is but one ~~point~~ chance to gaine a & 5 to gaine nothing at each cast, I make $b=0=c=r$. $p=1$ & $q=5$. Therefore by the

The first mans lot is worth $\frac{a}{6}$. The 2ds is worth $\frac{a}{6}-\frac{a}{36}=\frac{5a}{36}$. The Thirds is worth $\frac{5a}{36}-\frac{5a}{216}=\frac{25a}{216}$. The 4ths is $\frac{25a}{216}-\frac{25a}{1296}=\frac{125a}{1296}$ The fifts lot is worth $\frac{125a}{1296}-\frac{125a}{7776}=\frac{625a}{7776}$. The Sixts lot is $\frac{625a}{7776}-\frac{625a}{46656}=\frac{3125a}{46656}$. &c. Soe yt their lots are as 7776 : 6480 : 5400 : 4500 : 3750 : 3125

Soe yt if I cast a die two or more times tis 1 to 5 yt I cast a cise at ye first cast & 11 to 25 yt I throw it at two casts, & 91 to 125 yt I cast it at thrice, & 671 to 625 yt I cast it once in 4 trialls, & 4651 to 3125 yt I cast it once in 5 times. &c

3. If I bargaine to cast severall sorts of lots successively at ye same stake ye valor of each lot is thus found viz: The first prop: gives ye valor of ye first lot; wch valor being detracted from ye stake, ye remainder is ye stake of ye 2d lot wch therefore may bee also found by ye first prop: &c.

As if I gaine a by throwing 12 at ye first cast, or 11 at ye 2d or 10 at ye 3d &c wth two dice. Since at ye first cast there is but one chance for a (viz 12) & 35 for nothing Therefore its valor is $\frac{a}{36}$ (by Prop 1). & ye stake for ye 2d cast is $a-\frac{a}{36}=\frac{35a}{36}$. Now since there are two chances for ~~it~~ (viz ⁙⁙ ⁘ & ⁘ ⁙⁙) & 34 for 0 ~~at ye 2d cast~~ therefore its valor is $\frac{2\times 35a}{36\times 36}=\frac{35a}{648}$. & ye stake for ye 3d ~~chance~~ lot is $\frac{595a}{648}$ for wch there are 3 chances (viz ⁘ ⁘, ⁙⁙ ⁛, ⁛ ⁙⁙) & 33 for nothing Therefore its valor is $\frac{595a}{7776}$.

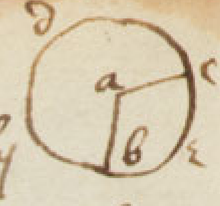
4 If I bargaine wth one or two more to cast lots in order untill one of us by an assigned lott shall win ye stake a: Since ye chances may succede infinitly I onely consider ye first revolution of them The valor of each mans whole Expectation being in such proportion ~~one~~ to another as ye valors of their lots in one revolution. & ye valor of each mans ~~first~~ lot being to ye valor of his whole expectation as ye summe of ye valors of their ~~first~~ lots to ye stake a.

As if I contend wth another yt who first throws 12 wth 2 dice shall have a, I having ye dice. ~~My~~ first lot is worth $\frac{a}{36}$ (by ~~[illegible]~~ prop 1), The 2d first lot is worth $\frac{35a}{36 \times 36}$. ~~And~~ $\frac{a}{36} : \frac{35a}{36 \times 36} :: 36 : 35 ::$ my Expectation: to his. for ye two first lots make one revolution because ~~I~~ I have ye same lot If I throw a 2d time yt I had at ye first. Therefore $\left(36+35=71 : a :: 36 : \frac{36a}{71}\right) \frac{36a}{71}$ is my interest in ye stake.

If or bargaine bee soe yt there is some lott at ye beginning of or play wch returnes not in ye ~~[illegible]~~ after revolutions, detract ye valor of those irregular lotts from ye stake & ye rest shall bee ye stake of ye ~~[illegible]~~ lots wch follow & revolve successively. As if I contend wth another yt who first casts 11 must have a, onely I have ye first cast for 12. My first lot is worth $\frac{a}{36}$. & ye stake for or after throws is $\frac{35a}{36}$. his first lot being $\frac{35a}{648}$. & my next lot $\frac{595a}{11664}$. soe yt his share in ye stake $\frac{35a}{36}$ is to mine as $\frac{35a}{648} : \frac{595a}{11664} :: 18 : 17$. Soe yt my share in it is $\frac{17a}{36}$. To wch adding ~~[illegible]~~ ye valor of my first lot viz: $\frac{a}{36}$, ye summe is $\frac{18a}{36} = \frac{a}{2}$, my interest in ye stake a at ye begining.

5 If ye Proportion of the chances for any stake bee irrationall ~~the~~ interest in the stake may bee found after ye same manner. As if ye Radij ab, ac, divide ye horizontall circle bcd into two pts

abec & abde in such proportion as 2 to $\sqrt{5}$. And if a ball falling perpendicularly upon ye center a doth tumble into ye portion abec I winn (a): ~~B~~ But if into ye other portion, I win b. my hopes is worth $\frac{2a+b\sqrt{5}}{2+\sqrt{5}}$.



Soe if a die bee not a Regular body but a Parallelipipedon or otherwise unequall sided, it may bee found how much one ~~side~~ cast is more easily gotten then another.

6 Soe yt ye facility of ye chances & ye stake belonging to each chance being knowne ye worth of the lott may bee ever found by ye precedent precepts. And if they bee not both immediatly ~~found~~ known they must bee sought before ye valor of ye lott can bee found.

As if I want two games at Irish & my adversary three to win a, & I would know ~~ye value of~~ my interest in ye stake (a.) my first lot ~~[illegible]~~ can gaine me nothing but ye advantage of another lot, & therefore to know its vallue I must first find ye value of yt other lot &c. ffirst therefore if wee each wanted one lot to win a or interest in it would bee equall viz my lot worth $\frac{a}{2}$. 2dly If I want ~~the lots~~ one games & my adversary two, & I gaine ye next game yn I gaine a but if I loose it I onely gaine an equall lot for a at ye next game wch is worth $\frac{1}{2}a$, Therefore my interest in ye stake is $\frac{a+\frac{1}{2}a}{2}=\frac{3a}{4}$. 3dly If I want one game & my adversary three & I gaine ye next game I get a; but if I loose it, then I want one game & my Adversary but two, yt is I get $\frac{3a}{4}$: Therefore (there being one chance for a & one for $\frac{3a}{4}$) my interest in ye stake is $\frac{a+\frac{3a}{4}}{2}=\frac{7a}{8}$. 4thly If I want 2 games & my adversary 3; & I win I get $\frac{7a}{8}$. but if I loose I get $\frac{1}{2}a$ for or chances

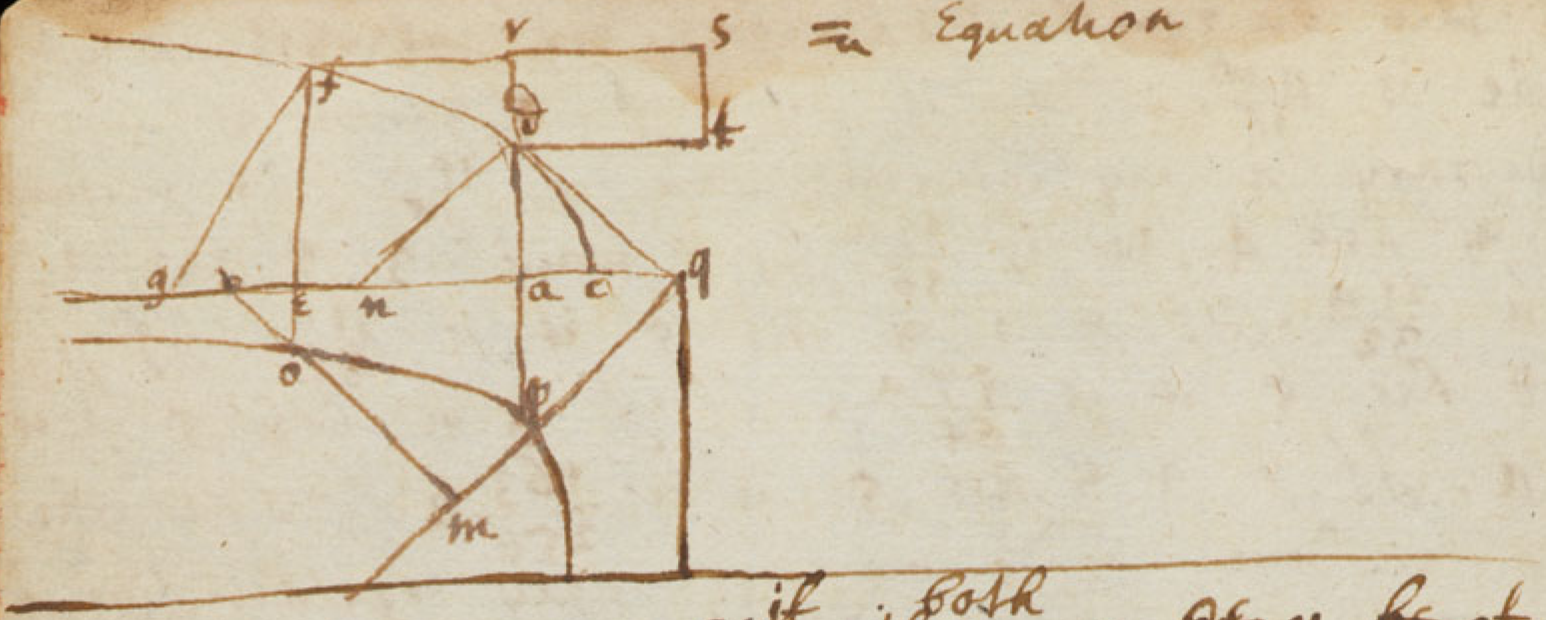
will then bee equall; Therefore my interest in ye stake is $\frac{11a}{16}$. Soe if I want 1 games & my adversary 4 my interest in a is $\frac{15a}{16}$. If I want two & hee 4, it is $\frac{13a}{16}$. If I want 3 & hee 4 it is $\frac{21a}{32}$. If I 1 & hee 5 it is $\frac{31a}{32}$. If I 2 & hee 5 it is $\frac{57}{64}a$. If I 3 & hee 5 it is $\frac{99}{128}a$. If I 4 & hee 5, it is; $\frac{163}{256}a$. (The like may bee done if 3 or more play together. (as if one wants one game, another 3 a third 4: Their lots are as 616; 82: 31. &c.) As also if their lots bee of divers sorts.)

By this meanes also some of ye precedent questions may bee resolved. as if I have two throws for a cise to win a, wth one die; If I have missed my first lot already, I have at my second cast five chances for nothing & one for a. therefore yt cast is ~~worth~~ $\frac{a}{6}$. Soe yt in my first cast I had five chances for $\frac{1}{6}a$ & one for ~~[illegible]~~ a, wch therefore ~~[illegible]~~ (wth my 2d cast) is worth $\frac{11}{36}a$. That is his ~~[illegible]~~ 11 to 25 ~~whither~~ yt I cast a cise ~~at~~ once in two throws. as before

By this meanes also my lot may bee known if I am to draw 4 cards of severall sorts out of 40 cards 10 of each sort.

Or if out of two white & 3 black stones I am blindfold to chose a white & black one.

= Equation



An equation given; if ~~either~~ both x, ~~or~~ y, ~~be of a~~ have divers ~~quan~~ dimensions, try if ye roote of one of ye may be extracted: ~~whereby ye = may often be diminished reduced to fewer termes & sometimes to fewer dimensions if it cannot ye line~~ & ~~..~~ If a quantity wherein y is not is divided by ~~x~~ in ye ~~equa~~ ~~..~~ line equall to x. yt crooked cannot be squared.

The line cdf is a Parab. $^4ac = {}^2ad = r = {}^2na = {}^2ge$. $ce = x$.

$ef = y$. $rx = yy$. $ge : ef :: \frac{1}{2}r : \sqrt{rx} :: eo : ap$.

$eo = z$. $ap = a$. $\frac{1}{2}ra = z\sqrt{rx}$. $\frac{1}{4}rraa = rzzx$. 93

$\frac{raa}{4} = zzx$. or supposing $ea = y$. $x = y + \frac{1}{4}r$ &

$\frac{raa}{4} = zzy + \frac{1}{4}zzr$. wch shews ye nature of ye crooked line po. now if $dt = ap$. yn $drst = eoap$. for supposing eo moves uniformely from ap, rs moves from dt with motion decreasing in ye same pportion yt ye line eo doth shorten. Suppos $aq = ap = \frac{r}{2} = a$ & $eq = y$. $x = y - \frac{1}{4}r$. then $\frac{r^3}{16} = zzy - \frac{1}{4}zzr$. suppose

$z = a + y$. yn $\frac{r^3}{16} = aax + 2ayx + yyx$. Or

$aax + 2ayx + yyx + \frac{1}{4}aar + \frac{1}{8}ayr + \frac{1}{4}yyr = \frac{r^3}{16}$. Or

$aax + 2ayx + yyx - \frac{1}{4}aar - \frac{1}{8}ayr - \frac{1}{4}yyr = \frac{r^3}{16}$. Or

suppose $z = y - a$, yn $\frac{r^3}{16} = aax - 2ayx + yyx$. Or

$aax - 2ayx + yyx + \frac{1}{4}aar - \frac{1}{8}ayr + \frac{1}{4}yyr = \frac{r^3}{16}$. Or

$aax - 2ayx + yyx - \frac{1}{4}aar + \frac{1}{8}ayr - \frac{1}{4}yyr = \frac{r^3}{16}$. Or,

if $x = a - y$. $\frac{r^3}{16} = zza - zzy$. &

$a^3 + {}^2aay + ayy - aax - 2ayx - yyx = \frac{r^3}{16}$. Or

$a^3 - {}^2aay + ayy - aax + 2ayx - yyx = \frac{r^3}{16}$.

$ob^2 = {}^2z^2$. $mp = \zeta$. $mq = a + \zeta = mb$. $mo = y = a + \zeta - \sqrt{2zz}$

$pq (= a) : aq (= \frac{1}{2}r) :: bq (= x + z) : mb = y + \sqrt{2zz}$

$ay + a\sqrt{2zz} = \frac{1}{2}rx + \frac{1}{2}rz$. $\frac{{}^2ay + {}^2z\sqrt{2aa}}{r} - \frac{1}{2}z = x =$

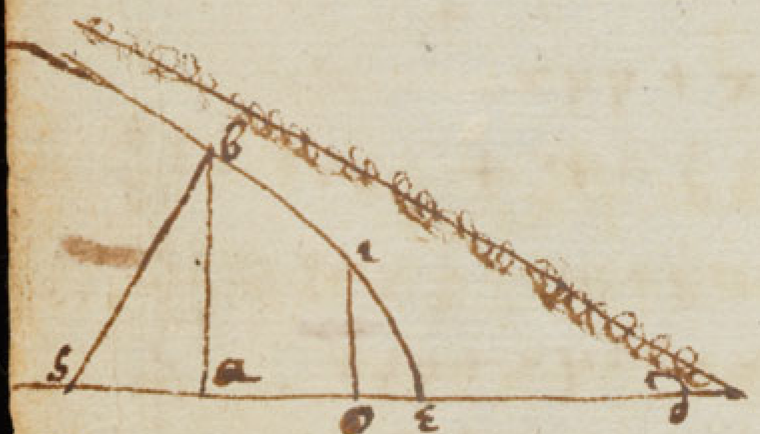
$\frac{r^3 + 4zzr}{16zz}$. $32zzay + 32z^3a\sqrt{2} - 8z^3r - r^4 - 4zzrr = 0$

$mv = \xi = a + \zeta - z\sqrt{2}$. $\frac{\xi - a - \zeta}{\sqrt{2}} = z$. $\frac{\xi^2 - {}^2\xi a - {}^2\xi\zeta + aa + {}^2a\zeta + \zeta\zeta}{2}$

$= zz$. $\frac{\xi^3 - 3\xi^2 a - 3\xi^2\zeta + 3aa\xi + 6a\zeta\xi + {}^3\zeta\zeta\xi - a^3 - 3aa\zeta - 3a\zeta\zeta - \zeta^3}{\sqrt{8}} = z^3$

$a\sqrt{2} + 32\sqrt{2aa} - 8r = c$

$cz^3 + 32a\zeta zz - r^4 = 0$
$- 4rr$


ς
β
α
ι
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δ

$$\begin{array}{llll}
c\xi^3 - 3ca\xi^2 & +3caa\xi & -ca^3 & \\
+32a\xi^3 - 3c\zeta\xi^2 & +6ca\zeta\xi & -3ca^2\zeta & \\
 & +3c\zeta\zeta\xi & -3ca\zeta^2 & \\
-64aa\xi^2 & +32a^3\xi & -c\zeta^3 & =0 \quad \text{Or} \\
-64a\zeta\xi^2 & +64aa\zeta\xi & -4rraa & \\
-4rr\xi^2 & +32a\zeta\zeta\xi & -8rra\zeta & \\
 & +8rra\xi & -4rr\zeta\zeta & \\
 & +8rr\zeta\xi & -r^4 &
\end{array}$$

$$\begin{array}{llll}
\xi^3 - \frac{d}{e}\zeta\xi^2 & +gg\xi & -f^3 & =0.. \\
-d & +k\zeta & -\zeta m^2 & \\
 & +\frac{i}{g}\zeta\zeta & -n\zeta^2 & \\
 & & -\frac{o}{n}\zeta^3 &
\end{array}$$

Let $\varepsilon d = a$. $\alpha\varepsilon = x$ $ab = y$. ~~$a\ldots$~~ $ss = v$. $sb = s$.

$ss = y^2 + vv + xx - 2vx$. ~~$\ldots$~~

~~$\ldots$~~ $aax + a^3 = yyx$

$y^2 = ss - vv + 2vx - xx.$

$aax + a^3 = ssx - vvx + 2vx^2 - x^3$

$$\begin{array}{llll}
x^3 - 2vx^2 & +vv\,x & +a^3 = 0 & \\
 & -ss & & \\
 & +aa & & \\
2 \quad 1 & 0 & -1 &
\end{array}$$

~~$2x^3 - 2vx^2 - a^3 = 0.$ $\frac{2x^3 - a^3}{2xx} = v$~~

$aax = yyx + yya.$ $y^2 = ss - vv + 2vx - xx$

$aax = ssx - vvx + 2vxx - x^3 + ssa + 2avx - axx - avv.$

$$\begin{array}{llll}
x^3 - 2vx^2 & -ss\,x & -ssa = 0. \\
+a & +vv & +avv \\
 & -2av & \\
 & +aa &
\end{array}
\qquad
\begin{array}{llll}
x^2 - 2\varepsilon x + \varepsilon\varepsilon & \times\; x + f & & \\
x^3 - 2\varepsilon x^2 & +\varepsilon\varepsilon x & +\varepsilon\varepsilon f = 0 & \\
+f & -2\varepsilon f x & &
\end{array}$$

$-2v + a = f - 2\varepsilon$. $f = a + 2\varepsilon - 2v$. $\varepsilon\varepsilon f = avv - ass$

$-\varepsilon\varepsilon a + 2\varepsilon^3 + 2\varepsilon\varepsilon v + avv = ss.$

$$\frac{\begin{array}{lll} vvx - 2vxx & +x^3 & -a^2 \\ vva - 2vax & +ax^2 & -x \\ & +aax & \end{array}}{a} \qquad \frac{\varepsilon\varepsilon v + avv - 2\varepsilon^3 - \varepsilon\varepsilon a}{a}$$

To Square those lines in w^ch is y onely

If y is in but one terme onely of y^e Equation (as $xx=ay$. or, $a^3=xxy$) resolve y^e Eq: into y^e proport $y:a$ (as $y:a::xx:aa$. or, $y:a::aa:xx$.)

If y^e line hath Assymptotes

$x^3=aay$. $v=\frac{3x^5}{a^4}+x$.

95

$a \quad vv - 2vx^2 + x^3 = 0 \qquad v = \dfrac{4x^3 + 2axx + aax + 2\frac{x^4}{a}}{4xx + 2ax + 2\frac{x^3}{a}}$

$x \quad -2vax + ax^2$

$-a \quad -2v\varepsilon\varepsilon + aax$

$-x \quad -\frac{3x\varepsilon\varepsilon v}{a} + 2\varepsilon^3 \qquad v = \dfrac{4ax^2 + 2aax + a^3 + 2x^3}{4ax + 2aa + 2xx}$

$+\varepsilon\varepsilon a$

$+\frac{2x\varepsilon^3}{a} \qquad Sa = \dfrac{4ax^2 + 2x^3 + 2aax + a^3 - 4ax^2 - 2x^3 - 2aax}{4ax + 2aa + 2xx}$

$+x\varepsilon\varepsilon$

$Sa = \dfrac{a^3}{2x^2 + 4ax + 2aa}$

$\dfrac{2a4x}{\cancel{\quad}}\ :\ \dfrac{2a6}{4xx + 8ax + 4aa} :$

$x + a$

$2aax : \dfrac{2a6}{8x^3 + 24ax + 24aax + 8a3} :: 2x^3 + 2ax^2$ ad.

$\dfrac{2a4x^2 + 2a5x}{8x^3 \&} \neq \dfrac{a4x}{4xx + 8ax + 4aa}$

$\cancel{x = \frac{44a}{aa - 44} \quad x^2 = \frac{44a^2}{a4 + 2a^2 + 44}}$

$\dfrac{a5\,44}{4aax^2 + 8a^3x + 4a^4 - 44 - 844a - 44aa} \qquad 2x^3 + 4ax^2 + 2aax - a^3 = 0$

$a - 44 \qquad 0\varepsilon = p.$

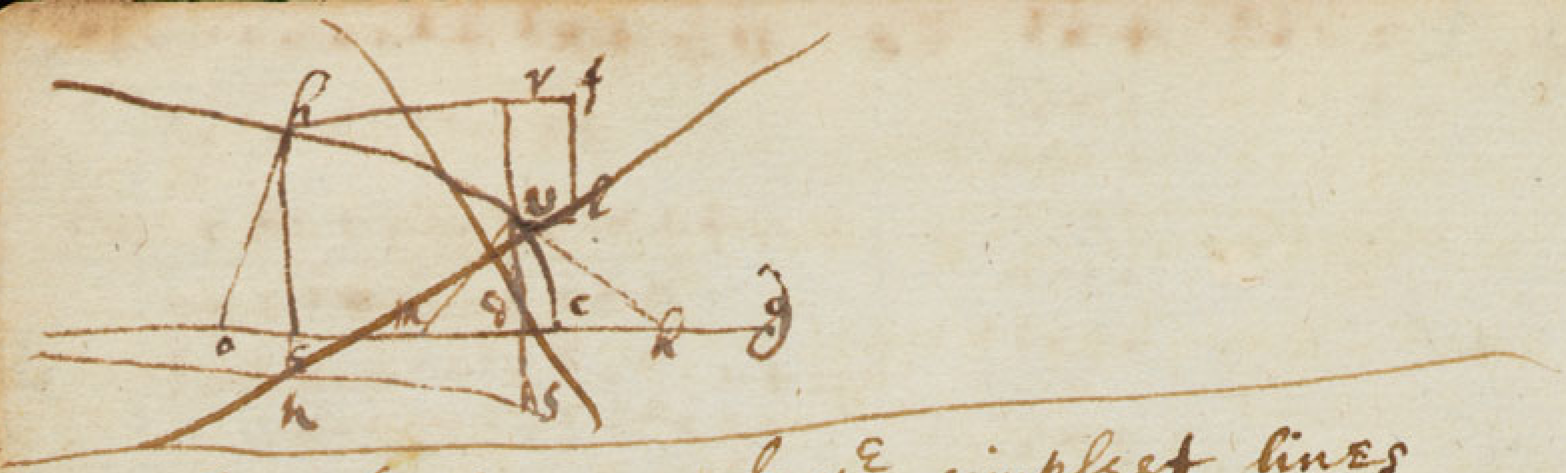
$\sqrt{\dfrac{aax}{a+x}} : \dfrac{a^3}{2x^2 + 4ax + 2aa} :: p : z$

$\dfrac{z^2aax}{a+x} = \dfrac{a6pp}{4x^4 + 16ax^3 + 8aaxx + 16aa + 16a^3x + 4a^4}$

$a5pp + a4xpp = 4zzx5 + 16zzax^4 + 24aazzx^3 + 16zza^3x^2 + 4a4z^2x$

Divided by $x + a$ it produceth.

$4zzx^4 + 12zzax^3 + 12zzaax^2 + 4zza^3x - a4pp = 0$



By y^e Squares of y^e simplest lines
to square lines more compound.

1st those wherin is y.

find y^e valor of y. If y^e number of y^e termes in y^e denom: thereof be neither 1.3.6.10.15.21.28. &c. y^e line cannot be squared
If it have but one terme tis squared by finding y^e square of each particular terme in y^e valor of y & y^n adding all those squares together. Example 1st.
$3x^4+a^4=yaxx$. & $y=\frac{3x^4+a^4}{axx}$. Then makeing y equall to each particular terme. $\frac{3xx}{a}=y$. $\frac{a^3}{xx}=y$ or $3xx=ay$ whose □ is $\frac{x^3}{a}$. & $a^3=xxy$. whose □ is $\frac{a^3}{x}$ Add these 2 □s together & they (viz: $\frac{x^4+a^4}{ax}$) are the □ of y^e line $3x^4+a^4=ayxx$. Againe
$2a^7 - 2bx^6 + x^7 = a^3x^3y$. Or $y=\frac{2a^7-2bx^6+x^7}{a^3x^3}$. y^n disjoynting y^e valor of y. $y=\frac{2a^4}{x^3}$. $y=\frac{x^4}{a^3}$. $y=\frac{-2bx^3}{aaa}$ Or $x^3y=2a^4$, whose □ is $\frac{a^4}{xx}$. $ya^3=x^4$, whose □ $\frac{x^5}{5a^3}$. $ya^3=-2bx^3$, whose □ $-\frac{x^4b}{2a^3}$. which 3 □'s (viz $\frac{10a^7+2x^7-5bx^6}{10a^3xx}$) ~~wh y^e □~~ taken together are y^e square sought for. And these lines may bee ever squared unlesse in y^e valor of y there bee found $\frac{aa}{x}$, $\frac{ab}{x}$, $\frac{ce+de}{x}$, &c. for y^e squareing of y^t line depends on y^e squareing of y^e Hyperbola. As in y^e line $xxxy = \frac{\cancel{\dots}}{x^4} + a^3x + a^4$.

$aax = y^3$. $cd = a$. $ce = x$. $eh = y$. $oc = v$. $ok = s$. 96

$ss = y^2 + x^2 - 2vx + v^2$. $x^2 = 2vx + ss - y^2 - v^2$.

$x = v - \sqrt{ + ss - y^2}$. $aav - aa\sqrt{ss - y^2} = y^3$.

$a^4v^2 - 2aavy^3 + y^6 - a^4ss + a^4y^2 = 0$

$0 \quad 3 \quad 6 \quad 0 \quad 2$

$6y^6 - 6aavy^3 + 2a^4yy = 0$. $v = \dfrac{a^4 + 3y^4}{3aay}$.

$\cancel{y : \dfrac{a^4+3y^4}{3aay} :: \dfrac{y^3}{aa} : \dfrac{a^4y^3 + 3y^7}{3a^4yy} = \dfrac{a^4y + 3y^5}{3a^4}}$

$v - x = \dfrac{a^4 + 3y^4}{3aay} - \dfrac{3y^4}{3aay}$ $\quad v - x = \dfrac{a^4}{3aay} = \dfrac{aa}{3y}$.

$y : \dfrac{aa}{3y} :: \dfrac{y^3}{aa} : \dfrac{aay^3}{3aayy} = \dfrac{y}{3}$. make $md = dv$ that is

$aa = 3yy$. $\dfrac{a}{\sqrt{3}} = y = md = dv$. $x = dc = \dfrac{a}{\sqrt{27}} = x$

$ds = dc = \sqrt{\dfrac{aa}{27}}$ $\quad \sqrt{c.\,aax} : \dfrac{aa}{3y} :: \sqrt{\dfrac{aa}{27}} : z = en$

$aax : \dfrac{a^6}{3aax} :: \dfrac{a^3}{27\sqrt{27}} : z^3$. $aaxz^3 = \dfrac{a^7}{81\sqrt{27}}$

$xz^3 = \dfrac{a^5}{81\sqrt{27}}$ wch expresseth ye nature of ye crooked line ns. $vl = ds$. $vlfr = dens$.

2dly. If it have 3 ~~termes. see~~ 6, 10, 19, 24, termes See if it may be reduced to {one or fewer} dimensions by adding or subtracting a knowne quantity to or from x. Example. $2bax + axx = bby + 2bxy + xxy$. wch (makeing $x + b = z$) is thus reduced

$zzy = - bba + azz$. Or $\dfrac{ - bba + azz}{zz} = y$

$aax + bby = y^3$. $x = v - \sqrt{ss - yy}$. $aav - aa\sqrt{ss - y^2} + bby = y^3$

$aav + bby - y^3 = aa\sqrt{ss - y^2}$.

$$\begin{array}{llllll} a^4v^2 & +2aavbby & -2aavy^3 & +b^4y^2 & -2bby^4 & +y^6 \\ -a^4ss & & & +a^4y^2 & & \\ 0 & 1 & 3 & 2 & 4 & 6 \end{array}$$

$$\frac{6y^5 - 8bby^3 + 2b^4y + 2a^4y}{-2aabb + 6aayy} = v$$

$$v - x = \begin{array}{l} -2b^4y + 6bby^3 - 6y^5 \\ +2b^4y + 2bby^3 + 6y^5 \\ +2a^4y - 8bby^3 \end{array} \qquad v - x = \frac{2a^4y}{6aay^2 - 2aabb}$$

$v - x = \dfrac{aay}{3y^2 - bb}$. $\dfrac{-bby + y^3}{aa} : \dfrac{aay}{3y^2 - bb} : \dfrac{y^3 - bby}{aa}$: &c ad.

$\dfrac{aay^4 - aabbyy}{3aay^3 - aabby}$. $\dfrac{y^3 - bby}{3y^2 - bb}$.

$\dfrac{aay}{3y^2 - bb} = \dfrac{y^3 - bby}{aa}$.

$a^4 = 3y^4 - 4bbyy + b^4$

$3y^4 = \underline{4bbyy - b^4 + a^4}$ $\quad a = b$

$\quad 3$

$y^2 = \dfrac{4bb}{3}$. $y = \dfrac{2b}{\sqrt{3}} = \partial m = \partial v$.

$\dfrac{8b^3}{3\sqrt{3}} - \dfrac{2bbb}{\sqrt{3}} - aax$. $\dfrac{2b}{3\sqrt{3}} = \partial c = \partial s$.

$y : \dfrac{aay}{3y^2 - bb} :: \dfrac{2b}{3\sqrt{3}} : z$. $yz = \dfrac{2aaby}{9y^2\sqrt{3} - 3bb\sqrt{3}}$

$9yyz - 3bbz = 2\dfrac{aab}{\sqrt{3}}$ An Equation expressing y^e^ nature of y^e^ line ns.

$aax + byx = y^3 . \quad x = v - \sqrt{ss - yy}$.

$aav + byv - y^3 = {aa \atop by} \sqrt{ss - yy}$

$a^4v^2 + 2aav^2by - 2aavy^3 + bbvvy^2 - 2bvy^4 + y^6$
$-a^4ss \qquad\qquad + a^4yy \quad + bb$
$\qquad\qquad\qquad - bbss$

${-2 \atop 0} \quad {-1 \atop 1} \quad {+1 \atop 3} \quad {0 \atop 2} \quad {2 \atop 4} \quad {4 \atop 6}$

$ss = 2a^4vv + 2aavvby + aavy^3 + 4bvy^4 - 2bby^4 - 4y^6$
$2a4$

$12a^4y^6 - 16a^4bvy^4 + 8a^4bby^4 + 8bbvvy^2 - 12abvy^3 + 4abvvby -$
$4a^8yy$

$12a^4y^6 - 16a^4bvy^4 \quad \cancel{+4\ldots} \quad \cancel{+4\ldots} \quad + 4abvvby$
$-8b^3vy^6 + 8a^4bby^4 \quad -12abvy^3 \quad +4a^8y^4$
$+4b^4y^6 - 2aabbvy^5 \quad -4aavvb^3 \quad \cancel{-4\ldots} \quad \}0$

$+8y^8bb$

$+4a^6by\,vv$
$-4aab^3y^3$ $= 12a^6y^3v \quad -8a^4bby^4$
$2aab^2y^5 \quad -8y^8bb$
$16a^4by^4 \quad -4b^4y^6$
$8b^3y^6 \quad +12a^4y^6$
$4a^6by - 4aab^3y^3$

$v = \dfrac{6a^6y^2 + a^2b^2y^4 + 8a^4by^3 + 4b^3y^5}{4a^6b - 4aab^3y^2} \; \mho \; \sqrt{\dfrac{36a^{12}y^4 + 8aab^5y^9 + 12a^8b^2y^6 + 64a^8bby^6 + 16a^{10}by^5 + 16b^6y^{10} + 24a^6b^3y^7 + a^4b^4y^8 - ss}{16a^{12}b^2 - 32a^8b^4y^2 + 16a^4b^6y^4}} \quad \dfrac{-8a^4bby^3 - 8y^7bb - 4b^4y^5 + 12a^4y^5}{4a^6b - 4aab^3y^2}$

$a3x = y^4. \quad x = v - \sqrt{ss - yy}. \quad a^3v - y^4 = a^3\sqrt{ss - yy}$

$a6v^2 - 2a^3vy^4 + y8 + a6yy. \quad v = \dfrac{4y^6 + a6}{4a3y2}$

$-a6ss \qquad 4 \qquad 8 \qquad 2$

0

$v - x = \dfrac{4y6 + a6 - 4y6}{4a3y^2} = \dfrac{a3}{4y^2}.$

$y. \dfrac{a3}{4y^2} :: \dfrac{y^4}{a3} : \dfrac{a3y^4}{4a3y3} = \dfrac{y}{4}$

$aax - aay = y^3$. $aav - aay - y^3 = aa\sqrt{ss - \cancel{yy}}$

$a^4v^2 - 2a^4vy - 2aavy^3 + 2a^4y^2 + 2aay^4 + y^6$
$-a^4ss$

$\quad 0 \quad 1 \quad 3 \quad 2 \quad 4 \quad 6$

$$v = \frac{3y^5 + 4aay^3 + 2a^4y}{a^4 + 3aayy}$$

$$v - x = \frac{3y^5 + 4aay^3 + 2a^4y - aay^3 - 3y^5 - ya^4 - 3aay^3}{a^4 + 3aayy}$$

$$v - x = \frac{aay}{aa + 3yy} . \; y : \frac{aay}{a^2 + 3y^2} :: \frac{a^2y + y^3}{aa} : \frac{a^4y^2 + a^2y^4}{a^4y + 3a^2y^3}$$

$$= \frac{aay + y^3}{aa + 3yy} \qquad \cancel{\frac{y^3 + aay}{aa} = \frac{aay}{aa + 3yy}}$$

$aax = by^2 + y^3.\quad aav - by^2 - y^3 = aa\sqrt{ss - yy}$

$a^4v^2 - 2aabyyv - 2aavy^3 + bby^4 + 2by^5 + y^6$ 101

$-a^4ss \quad + a^4$

$\quad 0 \qquad 2 \qquad\qquad 3 \qquad\qquad 4 \qquad 5 \qquad 6$

$v = \dfrac{3y^4 + 5by^3 + 2bby^2 + a^4yy}{2aab + 3aay}$

$v - x = \dfrac{3y^4 + 5by^3 + 2bby^2 - 2bby^4 - 3by^3 - 2by^3 - 3y^4 + a^4}{2aab + 3aay}$

$v - x = \dfrac{\cancel{by^3 + bby^2}\, a^4}{2aab + 3aay}$ $y : \dfrac{by^3 + bby^2}{aab + 3aay} :: \dfrac{by^2 + y^3}{aa} : aa$

$\dfrac{2bby^4 + by^5 + b^3y^3}{\cancel{aab} + 3aay} :$ $a = b.\quad a^3 + 3aay = byy + bby$

$aa + 3ay = yy - ay = 0 \quad yy = 2ay + aa$

$y = a + \sqrt{aa + aa}.\quad y = a + a\sqrt{2} = \partial m = v\partial.$

$\dfrac{a^3 + 3a^3\sqrt{2} + 6a^3 + 2a^3\sqrt{2} \quad a^3 + 2a^3\sqrt{2} + 2a^3}{aa} = 10a + 7a\sqrt{2} = \partial c$

$y : \dfrac{y^3 + by^2}{bb + 3by} :: 10b + 7b\sqrt{2} : z.\quad yz = \dfrac{10y^3 + 10by^2 + 7y^3\sqrt{2} + 7by^2\sqrt{2}}{b + 3y}$

$bz + 3yz = 10yy + 10by + 7yy\sqrt{2} + 7by\sqrt{2}.$

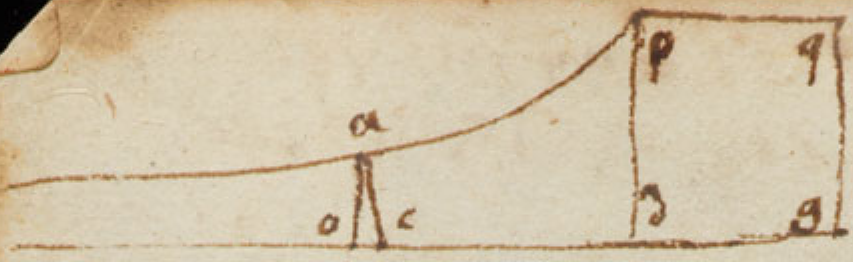
$axy = y^3 + b^3$. $ayv - y^3 - b^3 = ay\sqrt{ss - yy}$. 102

$$\begin{array}{lllll} aavvy^2 & -2avy^4 & -2avb^3y & +y^6 + 2b^3y^3 + b^6 \\ -aass & +aa & & \\ 0 & 2 & -1 & 4 \quad 1 \quad -2 \end{array}$$

$$v = \frac{4y^6 + 2b^3y^3 + 2aay^4 - 2b^6}{4ay^4 - 2ab^3y}$$

$$v - x = \frac{4y^6 + 2b^3y^3 + 2aay^4 - 2b^6 - 4y^6 + 2b^3y^3 - 4b^3y^3 + 2b^6}{4ay^4 - 2ab^3y}$$

$$v - x = \frac{ay^3}{2y^3 - b^3}.\ y : \frac{ay^3}{2y^3 - b^3} :: \frac{y^3 + b^3}{ay} : \frac{y^4 + b^4}{2y^3 - b^3}$$


a
o
c
p
q

$a3 = xyy \,.\; a = \partial g = \delta p \,.\; go = x \,.\; oa = y \,.\; cg = v \,.\; ca = s$

$ss = y^2 + xx - 2vx + v^2 \,.\; a^3 - xss + x^3 - 2vx^2 + vvx\,x$

$ \quad -1 \quad 0 \quad 2 \quad 1 \quad 0$

$\dfrac{2x^3 - a^3}{2x^2} = v \,.\; \sqrt{\dfrac{a3}{x}} : \dfrac{4x6 - 4a3x3 + a6}{4x4} :: x \,.$

$\dfrac{4x^6 - 4a^3x^3 + a6}{4xxa3} \,.$

$x - v = \dfrac{a3}{2x2} \,.\; \sqrt{\dfrac{a3}{x}} : \dfrac{a3}{2xx} :: p : z$

$\dfrac{zza3}{x} = \dfrac{a6pp}{4x4} \quad 4x^3zz = a3pp$

$go = y \,.\, oa = x$ &c: $a^3 = xyy \,.\; ss = xx + yy + vv - 2vy$

$xx = ss - yy + 2vy + vv \,.$

$a6 - ssy4 + y6 - 2vy5 + vvy^4 = 0$

$-4 \quad 0 \quad 2 \quad 1 \quad 0$

$v = \dfrac{2y6 - 4a6}{2y5} \quad y - v = \dfrac{2a6}{y5}$

~~$yy = ss - xx + vx$~~

$\dfrac{a3}{y2} : \dfrac{2a6}{y5} :: p : z \quad z = \dfrac{2a3p}{yyy} \,.\; zy^3 = 2a^3p \,.$

wch shewes ye nature of another crooked line yt may be squared.

$axx = y^3 . \quad x = v - \sqrt{ss - yy} . \quad x^2 = v^2 - 2v\sqrt{ss - y^2} + ss - y^2 .$

$av^2 + ass - ay^2 - y^3 = 2av\sqrt{ss - yy} .$

$a^2v^4 \cancel{+2a^2v^2s^2} \cancel{-2a^2v^2y^2} - 2av^2y^3 + aay^4 - 2ay^5 + y^6$

$+ aas4 \quad - 2aassy^2 - 2assy^3$

$-\frac{3}{}a^2v^2ss \quad \cancel{+2v^2a^2y^2}$

$+ 2vv\,aay^2 \quad -0 \quad -1 \quad -2 - 3$

$+3$

$3a^2s4 - 2a^2y^2ss + 3a^2v4 + 2v^2a^2y^2 - a^2y^4 + 4ay^5 - 3y^6$

$-6aavv$

$ss = \frac{2yy}{3} + \frac{6vv}{3} + \sqrt{\frac{4y4}{9} - \frac{v4 + y4}{3} \frac{+y^6 - 2ay^5}{3aa}}$

$$ss = \frac{2yy + 6vv}{3} - \sqrt{\frac{4y^4a^2 + 24aayyvv + 36aav4 - 9aav4 - 6aayyvv - 12ay^5 + 3aay^4 + 9y^6}{9aa}}$$

$$ss = \frac{2y^2 + 6v^2}{3} - \sqrt{\frac{9y^6 - 12ay^5 + 7aay^4 + 18aavvy^2 + 27aav4}{9aa}}$$

$a = b-A = C-B = D-\partial = \varepsilon-D = F-E = G-f = 1{,}117313.$

$A-a = \partial-C = ff-F = 0{,}921787.$ $B-b = E-\varepsilon = 0{,}706724.$

$A = b-a = \partial-B = D-C = f-E = G-F = 2{,}039100.$

$B-A = C-b = E-D = F-\varepsilon = 1{,}824037.$ $\varepsilon-\partial = aa-f = 2{,}234626.$

$b = D-B = \varepsilon-C = G-E = aa-F = AA-f = 3{,}156413.$

$C-A = E-\partial = F-D = 2{,}941350.$ $B-a = \partial-b = f-\varepsilon = 2{,}745824.$

$B = C-a = \partial-A = D-b = E-C = f-D = G-\varepsilon = 3{,}863137.$

$\varepsilon-B = aa-E = bb-f = 4{,}273726.$ $F-\partial = 4{,}058663.$ $A^2-F = 4{,}078200.$

$C = D-A = \varepsilon-b = E-B = F-C = f-\partial = G-D = aa-\varepsilon = B^2-f = 4{,}980450.$

$A^2-E = bb-F = 5{,}195513.$ $\partial-a = 4{,}784924.$

$\partial = D-a = f-C = A^2-\varepsilon = B^2-F = 5{,}902237.$ $bb-\varepsilon = 6{,}312826.$

$\varepsilon-A = F-B = G-\partial = aa-D = C^2-f = 6{,}097763.$ $E-b = 5{,}687174.$

$D = \varepsilon-a = f-B = G-C = A^2-D = bb-\varepsilon = B^2-E = C^2-F = \partial\partial-f = 7{,}019550.$

$E-A = F-b = 6{,}804487.$ $aa-\partial = 7{,}215076.$

$\varepsilon = G-B = aa-C = A^2-\partial = bb-D = C^2-E = D^2-f = 8{,}136863$

$E-a = f-b = B^2-\varepsilon = 7{,}726274.$ $F-A = 7{,}921800.$ $\partial\partial-F = 7{,}941337.$

$E = F-a = f-A = G-b = B^2-D = C^2-\varepsilon = 8{,}843587.$

$A^2-\partial = \partial\partial-E = D^2-F = 9{,}058650.$ $aa-B = bb-\partial = \varepsilon\varepsilon-f = 9{,}254176.$

$F = G-A = aa-b = B^2-\partial = C^2-D = E^2-f = 9{,}960900.$

$A^2-B = bb-C = D^2-E = \varepsilon\varepsilon-F = 10{,}175963.$ $f-a = \partial\partial-\varepsilon = 9{,}765374.$

$f = G-a = A^2-b = B^2-C = \partial\partial-D = D^2-\varepsilon = E^2-F = 10{,}882687.$

$aa-A = C^2-\partial = F^2-f = 11{,}078213.$ $B^2-C = \varepsilon\varepsilon-E = 11{,}293276.$

This table shews ye distance of any two notes As ye distance of C & E is B, or a 3d, or 3,863137 halfe notes Of B & E tis a 4th, or 4,98045 halfe notes. of B & F tis 6,097763 halfe notes, or greater yn a 5t b, by 0,095526 halfe notes &c.

$aa = xy.\ eg = eh = a.\ ga = x.\ ad = y.\ \cancel{dc} = s.\ cg = v$

$yy = ss - xx + 2vx - vv.$

$$\begin{array}{l} -a^4 + x^2s^2 - x^4 + 2vx^3 = 0 \\ \qquad\quad - x^2v^2 \\ +2 \qquad\ \ 0 \qquad -2 \qquad\quad -1 \end{array} \qquad \frac{2x^4 - 2a^4}{2x^3} = v$$

$$x - v = \frac{a^4}{x^3}.\ \frac{aa}{x} : \frac{a^4}{x^3} :: p : z.\ zxx = a^2p.$$

$8^{th} - 5^t = 4^{th} = G - D = C = 6^t - 3^d = E - B$

$5^t - 4^{th} = 5^t + 5^t - 8^{th} = 2^d = A.$

$4^{th} + 4^{th} = 8^{th} + 8^{th} - 5^t - 5^t = 7^{th} = F.$

$4^{th} - 3^d = 2^d\flat = a$

$8^{th} - 3^d = 6^t\flat = e$

$4^{th} + 3^d = 6^t = E$

$5 - 3^d = 3^d = 8^{th} - 6^t = b$

$3^d + 5^t = 7^{th}\sharp = f$

$7^{th}\sharp - 4^{th} = 2^d + 3^d = 5^t\flat = d = 3^d + 5^t - 4^{th} = -2^d\flat + 5^t.$

By ye helpe of concordant notes all ye notes in the ~~Gam ut~~ may bee thus tuned viz:

ffirst tune ye 8ths, G, G^2, G^3, G^4 &c.

Seacondly tune 5ts to ym both above ym D, D^2, D^3, D^4 & below them ^2C, C, C^2, C^3.

Thirdly tune 3ds to ym both above ym B, B^2, B^3, B^4 & below them $^2E\flat$, $E\flat$, $E^2\flat$, $E^3\flat$.

ffourthly from each B, B^2, B^3, B^4 rise a fift for $F\sharp$, $F^2\sharp$, $F^3\sharp$, $F^4\sharp$ & fall a 5t for ^2E, E, E^2, E^3.

ffiftly from $^2E\flat$, $E\flat$, $E^2\flat$, $E^3\flat$ rise a fift for $B\flat$, $B^2\flat$, $B^3\flat$, $B^4\flat$. & fall a fift for $^2A\flat$, $A\flat$, $A^2\flat$, $A^3\flat$.

Sixtly from D, D^2, D^3, D^4 rise a fift for A^2, A^3, A^4, A^5. & from ^2C, C, C^2, C^3 fall a fift for ^3F, ^2F, F, F^2.

Seaventhly from each $F\sharp$, $F^2\sharp$, $F^3\sharp$, $F^4\sharp$. rise a fift for $D^2\flat$, $D^3\flat$, $D^4\flat$, $D^5\flat$. The rest as A, $D\flat$ are supplyed by 8ths viz to A^2, $D^2\flat$ &c.

November 20. 1665.

$\frac{1}{2}$	360	2,55630247	2,5563025	360,000000	12,00000	G
$\frac{8}{15}$	384	2,58433118	2,5813883	381,40[illegible]6678.	10,882687	fl.
$\frac{9}{16}$	405	2,60745497	2,6064742	404,086406.	9,96090,0	F
$\frac{3}{5}$	432	2,63548364	2,6315600	428,114581.	8,843587.	E
$\frac{5}{8}$	450	2,65321247	2,6566458	453,571578.	8,136863.	ε
$\frac{2}{3}$	480	2,68124123	2,6817317	480,542367.	7,019550	D
$\frac{32}{45}$	512	2,70926992	2,7068175	509,1168824543	5,902237.	d
$\frac{3}{4}$	540	2,73239371	2,7319033	539,390559	4,980450	C
$\frac{4}{5}$	576	2,7604224	2,7569892	571,464474.	3,863137.	B
$\frac{5}{6}$	600	2,7781512[illegible]	2,7820750	605,445467.	3,156413.	b
$\frac{8}{9}$	640	2,80617993	2,8071608	641,446973.	2,039100	A
$\frac{15}{16}$	675	2,8293038	2,8322467	679,5895149.	1,117313	a
1	720	2,8573325	2,8573325	720,000000.	0,000000	G
How ye string 1 or 720 is to bee divided yt it may sound all ye musicall notes & halfe notes in an Eight		The proportion wch those musicall notes & ½ notes beare one to ye other (viz ye logarithmes of ye string sounding them)	Twelve exact or equidistant ½ notes (or ye logarithms of a cord divided into 12 geometricall parts) ye distance of each ½ note being 0,025085833333 &c. 1 just note being 0,050171666666 &c.	A string (720) divided into 12 (geometrically progressionall) parts, yt it may sound ye 12 exact ½ notes in an Eight	The proportion of all ye 12 musicall ½ notes in a Eight, an exact halfe note being a unite.	

~~$a^4 = xxyy. \; a^4 \ldots x^2 + vv x^3 - x^4 = 0$~~

~~$-vv$~~ $\quad 0 \quad -1 \quad -2$

$\cancel{\dfrac{x^4 - a^4}{x^3} = v. \; x - v = \dfrac{a^4}{x^3}}$ (263548369)

$a^4 = xy^3 \quad a8 = x. \; -a^8 + y^6 ss - y^8 + 2vy^7 - vvy^6 = 0$

$\quad +6 \quad 0 \quad -2 \quad -1 \quad 0$

$\dfrac{2y^8 - 6a^8}{2y^7} = v. \quad v - y = \dfrac{3a^8}{2y^7}.$

$\dfrac{a^4}{y^3} :: \dfrac{3a^8}{2y^7} :: p = \dfrac{2a^4}{y^3} = \dfrac{3a^8 p}{3y^7}. \; 2y^4 = \dfrac{3a^4 p}{2}$

0,921787 = A − a = ∂ − C = f − F. 2,039100 = A = b − a = ∂ − B = D − C = f − E = G − F

F − ε = 1,824037 = B − A = C − b = E − D. ε − ∂ = 2,234626

aa − f = 2,234626

0,706724 = B − b = E − ε.

bb − f

aa − E = 4,273726 = ε − B. F − ∂ = 4,058663

2,749824 = B − a = ∂ − b = f − ε.

4,078200 = A^2 − F

2,941350 = C − A = E − ∂ = F − D.

3,156413 = b = D − B = ε − C = G − E = aa − F = AA − f. bb − ε = 6312826

3,863137 = B = C − a = ∂ − A = D − b = E − C = f − D = G − ε

4,980450 = C = D − A = ε − b = E − B = F − C = f − ∂ = G − D = aa − ε = B^2 − f

4,784924 = ∂ − a

5,195513 = A^2 − E = bb − F

5,902237 = ∂ = D − a = f − C = A^2 − ε = B^2 − F. 7,9218 = F − A

6,097763 = ε − A = F − B = G − ∂ = aa − D = C^2 − f. 7,941337 = ∂∂ − F

6,687174 = E − b

6,804487 = E − A = F − b

7,099550 = D = ε − a = f − B = G − C = A^2 − D = bb − ε = B^2 − E = C^2 − F = ∂∂ − f

7,215076 = aa − ∂. 7,726274 = E − a = f − b = B^2 − ε

8,136863 = ε = G − B = A^2 − C = A^2 − ∂ = bb − D = C^2 − E = D^2 − f

8,843587 = E = F − a = f − A = G − b = B^2 − D = C^2 − ε

9,254176 = aa − B = bb − ∂ = εε − f. D^2 − F = 9,058650 = A^2 − ∂ = ∂∂ − E.

9,765374 = f − a = ∂∂ − ε

10,175963 = A^2 − B = bb − C = D^2 − E = εε − F

11,078213 = aa − A = C^2 − ∂ = F^2 − f

11,293276 = B^2 − C = εε − E

perhaps ∂ = E − b is better y^n ∂ = 3^d + 5^t − 4^th.

0	1,1	2	3,1	3,9	5	5,9	7	8,1	8,9	10	10,9	12
G	a	A	b	B	C	∂	D	ε	E	F	f	G
0	1,2	2	3,2	3,8	5	5,8	7	8,2	8,8	10	10,8	12
0	6	10	16	19	25	29	35	41	44	50	54	60

2,00000	g	2 = 2,0000.
1,88774		$\frac{15}{8}$ = 1,8750.
1,7818	a	$\frac{16}{9}$ = 1,777777 &c
1,6818		$\frac{5}{3}$ = 1,66666 &c
1,5874	b	$\frac{8}{5}$ = 1,6000
1,4983	c	$\frac{3}{2}$ = 1,5000
$\frac{64}{45}$=14222 1,4142136		$\frac{10}{7}$ = 1,4285714285714 &c
1,3349	d	$\frac{4}{3}$ = 1,33333 &c
1,2599		$\frac{5}{4}$ = 1,2500.
1,18920	e	$\frac{6}{5}$ = 1,2000.
1,12245	f	$\frac{9}{8}$ = 1,1250.
1,05946		$\frac{16}{15}$ = 1,066666 &c
1,00000	g	1 = 1,0000.
A string divided in a Geometricall progression.		A string divided by a Musicall progression

By this table it may appeare that a

Second minor ~~is higher~~
Second
Third minor } is higher by ye { 17th+, 59th+, 13th− }
Third
Fourth } is lower by ye { 15th−, 102th }
Fift mine is lower by ye 20th½ or higher by ye 20th½
~~Fift~~
Sixt minor } is higher by ye { 102th, 15th }
Sixt
Seaventh
Eight minor } is lower by ye { 13th, 59th, 17th }

parts of a note then it would bee were ye musicall chord divided in Geometricall progression

$\frac{1}{2}$ notes. notes
=1,058522.
=2,02.
=3,07
=3,93
4,99.
5,95.
6,05.

$1, \frac{15}{16}, \frac{8}{9}, \frac{5}{6}, \frac{4}{5}, \frac{3}{4}, \frac{18}{25}, \frac{5}{7}, \frac{32}{45}, \frac{45}{64}, \frac{2}{3}, \frac{5}{8}, \frac{3}{5}, \frac{9}{16}, \frac{8}{15}, \frac{1}{2}$

$\frac{24}{25}, \frac{7}{8}, \frac{6}{7}, \quad \frac{7}{10}, \quad \frac{7}{12}, \frac{4}{7}, \frac{25}{48}$

$\frac{20}{21}, \frac{9}{10}, \quad \frac{25}{36}, \quad \frac{5}{9}, \frac{21}{40}$

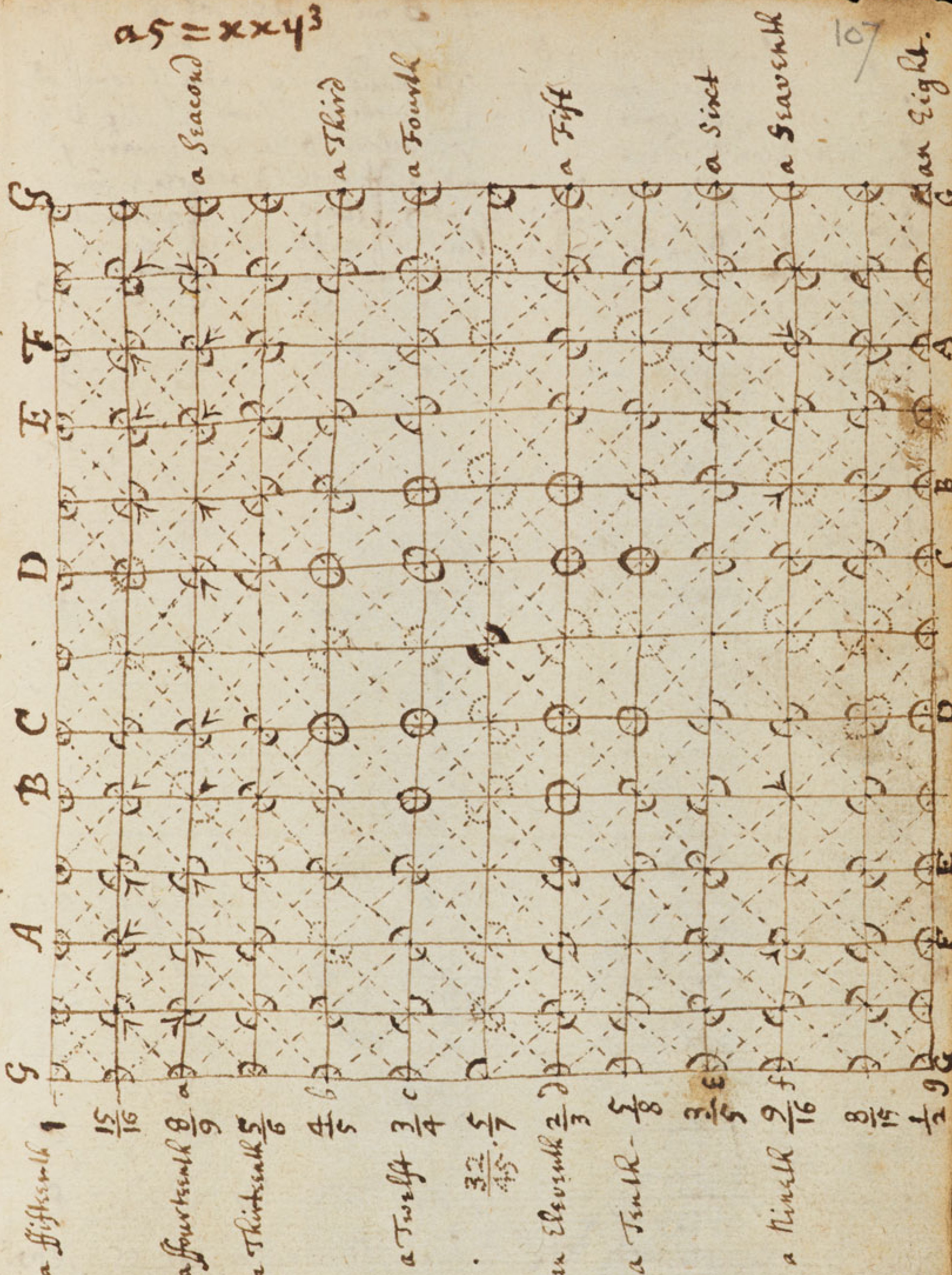
A 3d mine is higher yn just, by ye $\frac{5}{64+}$th pts of a note

A fift minor is h yn just by ye $\frac{3}{41}$th pts of a note

$\frac{1}{2}, \frac{15}{32}, \frac{4}{9}, \frac{5}{12}, \frac{2}{5}, \frac{3}{8}, \frac{16}{45}, \frac{1}{3}, \frac{5}{16}, \frac{3}{10}, \frac{9}{32}, \frac{4}{15}, \frac{1}{4}$

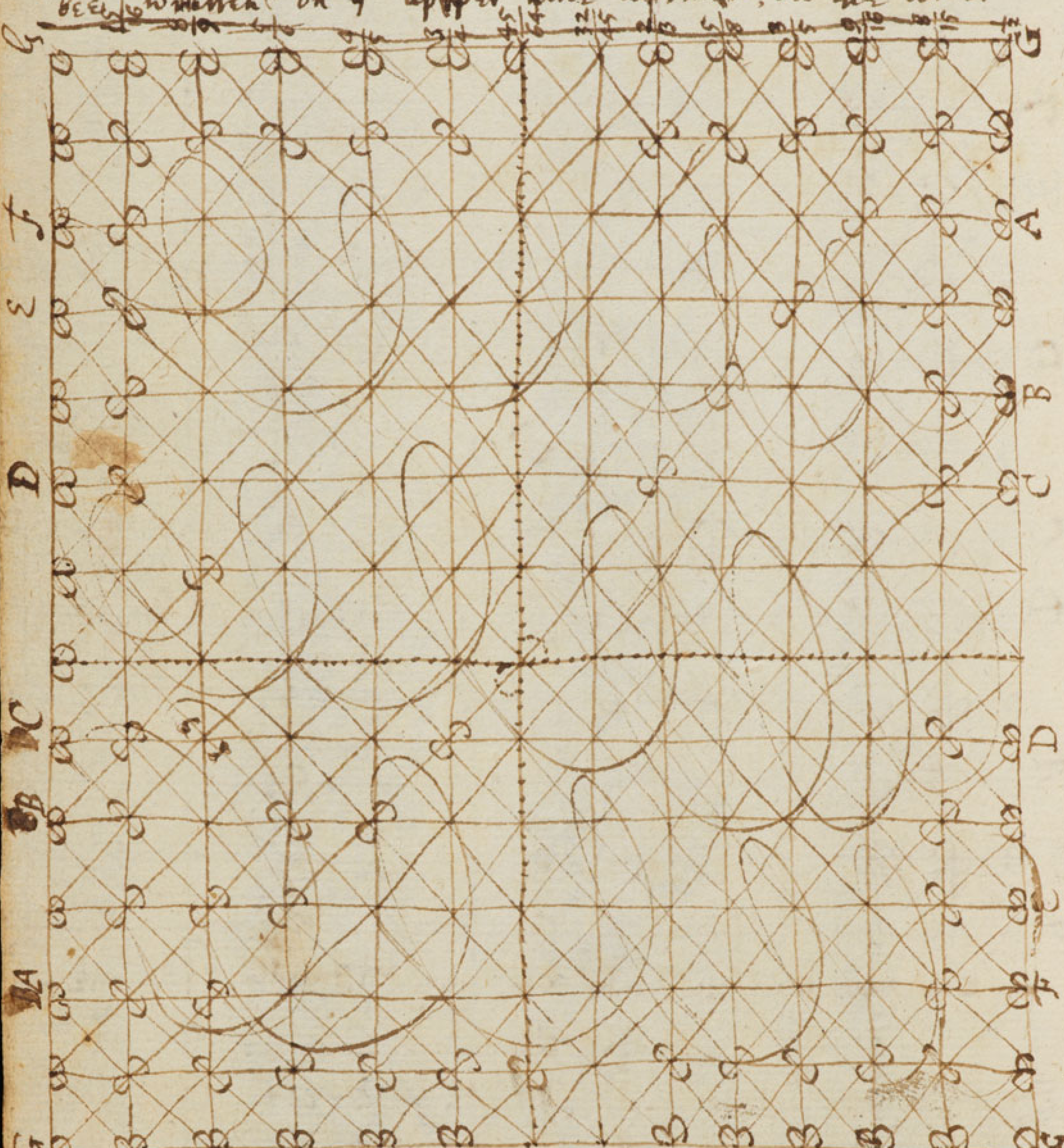
$\frac{1}{4}, \frac{15}{64}, \frac{2}{9}, \frac{5}{24}, \frac{1}{5}, \frac{3}{16}, \frac{8}{45}, \frac{1}{6}, \frac{5}{32}, \frac{3}{20}, \frac{9}{64}, \frac{2}{15}, \frac{1}{8}$

$a5 = xxy^3$



By this table may bee knowne ye distance of any ~~two~~ notes whither a true second of ye lesse, second, third of ye lesse, a third fourth &c: As to know ye distance twixt A re & D sol re I follow ye pricked stroke from A to D or from D to A where I find it crossed by a black crooked line as ~~four~~ against it, a fourth written, therefor I conclude A re & D la sol distant a true fourth.

And thus to find ye distance of B mi & D la sol re I follow ye prick line from ye top B to ye right hand side thence to ye bottom B thence towards ye left hand side untill I come over D. Or (wch is ye same) I follow ye prickt line from ye top D to the left hand side thence to ye bottom D, thence toward ye right hand side untill I come just over B, where I find ye pricked line to be crossed by a 3 strokes & against it to been written on ye upper line a tenth, on the lower



a ninth therefore tis a tenth minor, exactly. but if it

Eight

$a^3 = xyy$. $x = v + \sqrt{ss-yy}$. $a^3 - vyy = yy\sqrt{ss-yy}$ 108 ~~30~~

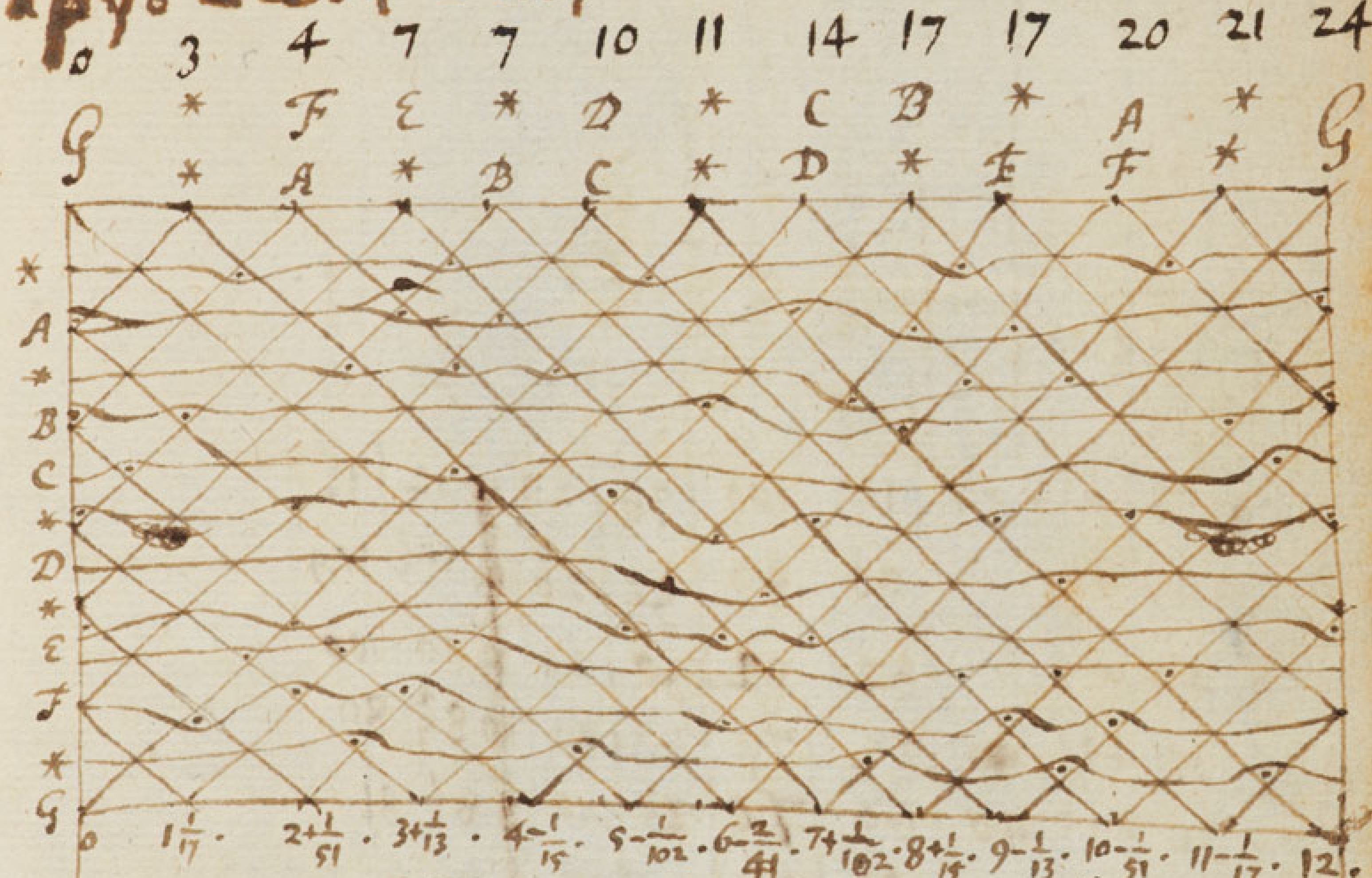
$a^6 - 2a^3vyy + \begin{matrix} vv \\ -ss \end{matrix} y^4 + y^6 = 0$

$-4 \quad -2 \quad 0 \quad 2$

$\dfrac{-2y^6+4a^6}{2a^3y^2}$. $x - v = \dfrac{2y^6-2a^6}{2a^3y^2}$.

$2a^3y^3 : 2y^6-2a^6 : p : z \qquad 2a^3y^3z = 2y^6p - 2a^6p$

$py^6 - a^3y^3 - a^6p$



0. 5. 9. 14. 16. 21. 25. 30. 35. 37 42. 46. 51.

0. 5. 9. 14. 17. 22. 26. 31. 36. 39. 44. 48. 53.

G. a. A. b. B. C d D e E F f G.

0. 4. 7. 11. 13. 17. 20. 24. 28. 30. 34. 37. 41.

0. 3. 5. 8. 9. 12. 14. 17. 20. 21. 24. 26. 29.

0. 4. 6. 10. 11. 15. 17. 21. 25. 26. 30. 32. 36.

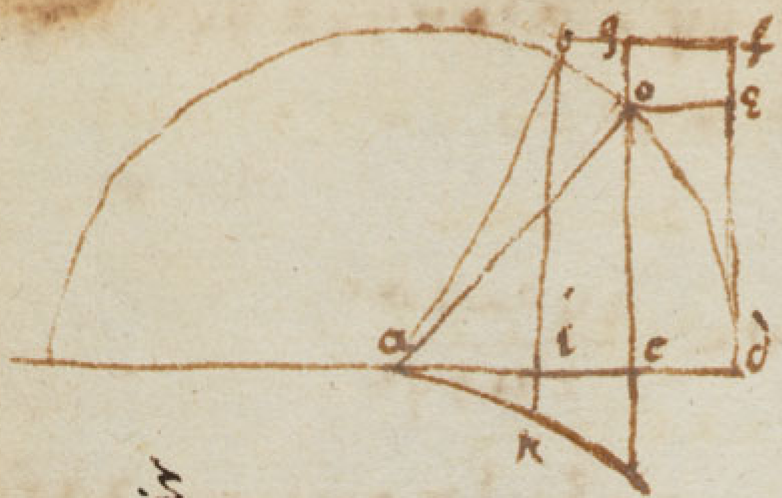
0. 11. 20. 31. 39. 50. 59. 70. 81. 89. 100. 109. 120.

0. 4 2. 6. 1: 5. 3. 7. 11. 6. 10. 8. 12

~~0. 1. 3. 4. 7. 8. 10. 13.~~

0. 2. 5. 7. 8. 10. 13. 15. 17. 18. 20. 23. 25.

0. 4. 4. 5. 7. 8. 11. 12. 13. 15. 16. 19. 20



o. a. . b. b+a.

o. a, a+b, a+b+c, 2a+b+c, 3a+2b+c

The notes	The proportion of their sounds	or thus	Or thus	How they may bee otherwise distinguished by figures thus	thus	or thus			Or thus	Or thus.
g	53	612	59	100	100	36	29	~~1~~	12	12.
f	48	555		90	89	32	26	[illegible]	$10\frac{2}{3}$	10,9
F	44	508		82	82	30	24	[illegible]	10	100
E	39	451	[illegible]	72	71	26	21	[illegible]	$8\frac{2}{3}$	8,9
e	36	415	40	69	70	25	20	[illegible]	$8\frac{1}{3}$	8,1
D	31	358		59	59	21	17	[illegible]	7	7
d	26	301		49	48	17	14	[illegible]	$5\frac{2}{3}$	5,9
C	22	254		41	41	15	12	[illegible]	5	5
B	17	197	19	31	30	11	9	~~4~~	$3\frac{2}{3}$	3,9
b	14	161		28	29	10	8	[illegible]	$3\frac{1}{3}$	3,1
A	9	104		18	18	6	5	~~3~~	2	2
a	5	57		10	11	4	3	~~2~~	$1\frac{1}{3}$	1,1
G	0	0	0	0	0	0	0	~~0~~	0	0

$ao=a=ad.\ dc=p.\ ai=x.\ oi=y.\ aa-xx=yy$

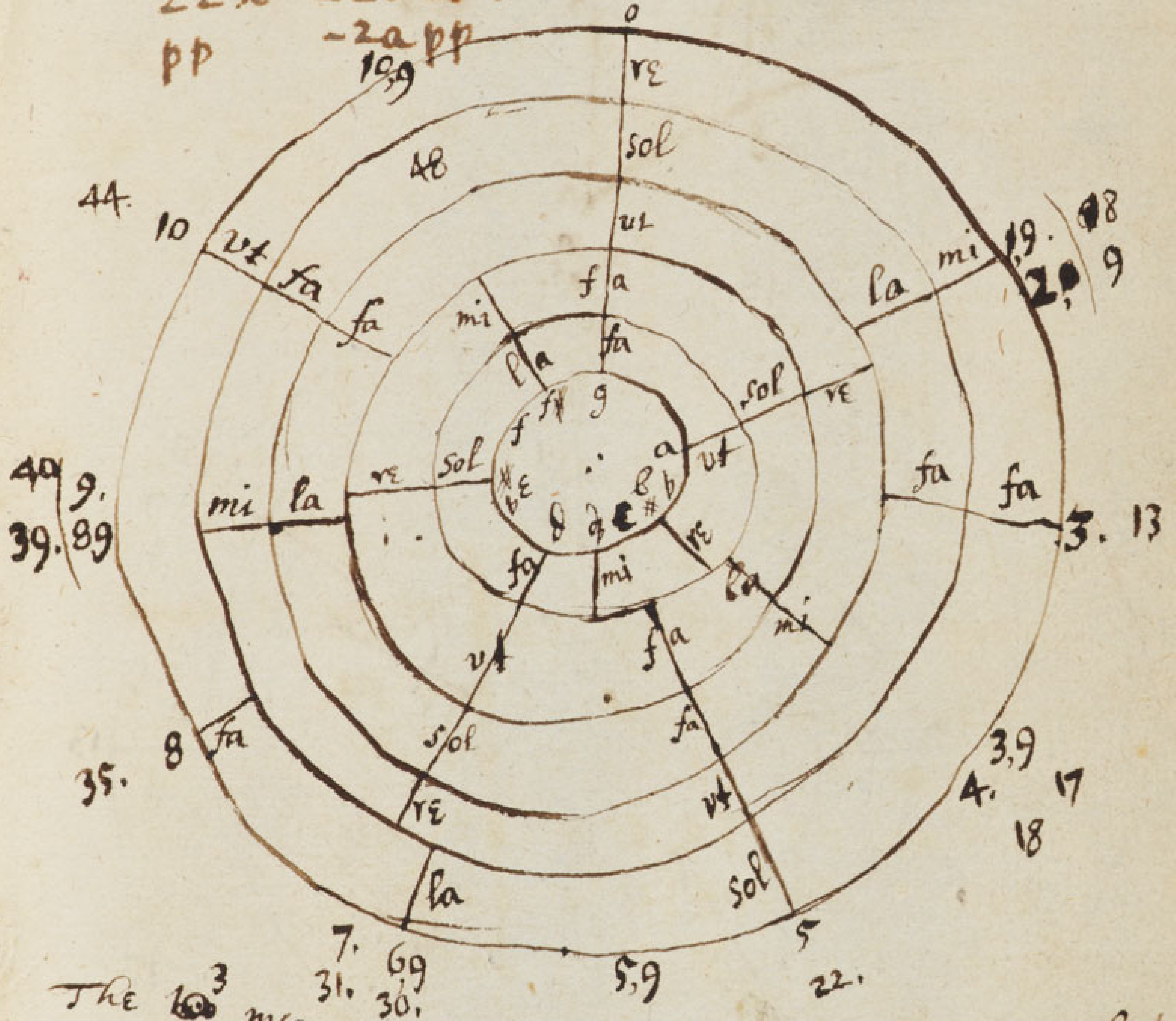
$in=z.\ xx:aa-xx::pp:zz.\ zzxx=aapp-p^2x^2.$

$id=x.\ oi=2ax-xx.$ ~~$aa-x^2xx$~~

$aa-2ax+x^2:2ax-x^2::pp:zz$

$zzx^2-2azzx+aazz=0.$

$pp\qquad -2app$

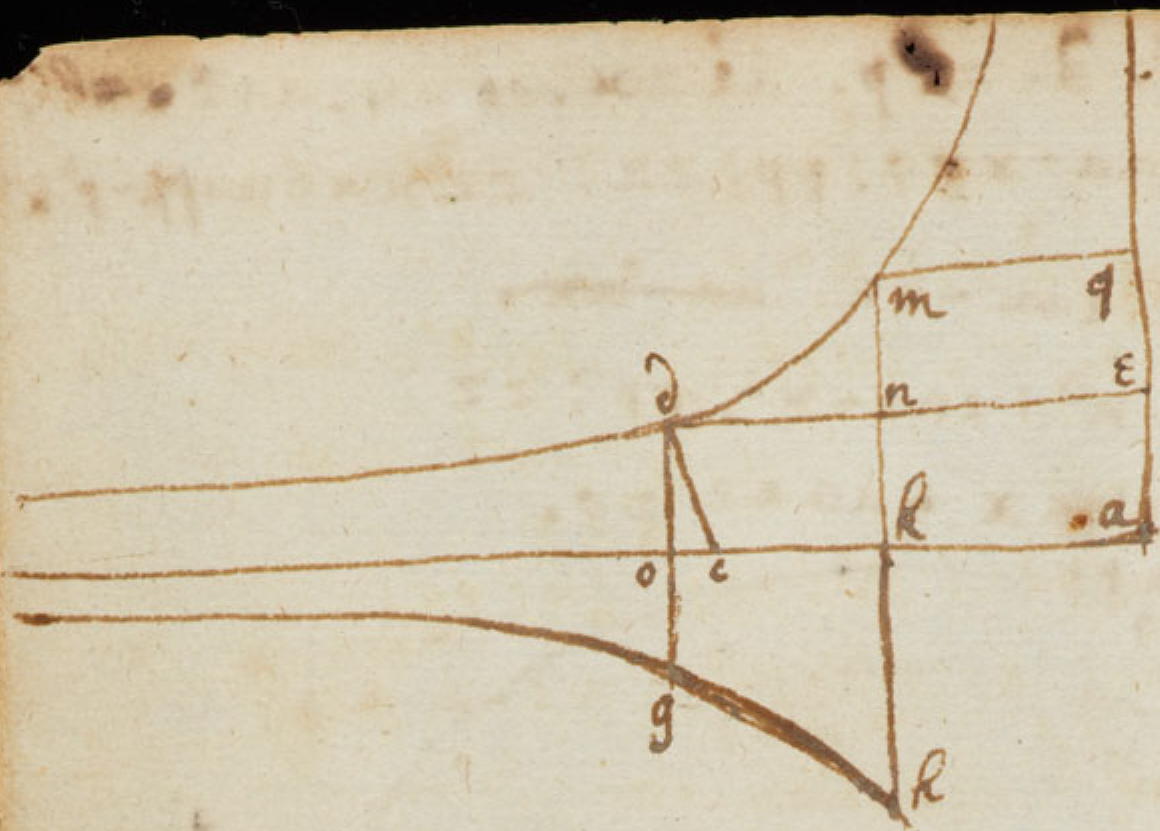


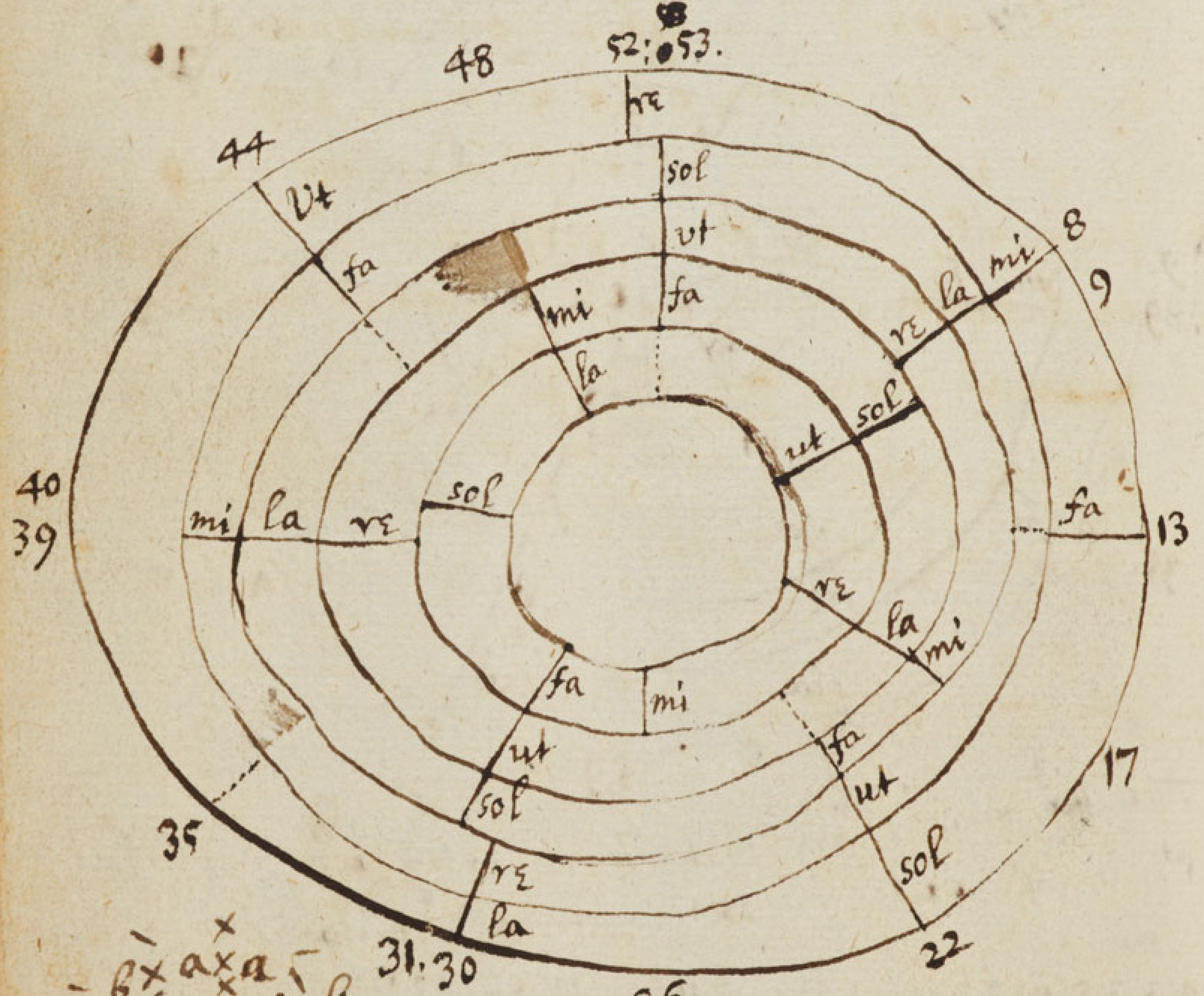
The ~~10~~ 3 meanes are best there being an imperfect 5t in ye outward Extreame & a tritonus in ye inmost.

1.6.4.3. 2.5/10.7.12.5. . 6.8.4. 6.11.8. 3.10.2. 3.9.10.12.7.

1. 6. 11. 8. 4. 3:9:10:12. 7. 2. 5.

109


m
q
n
ε
∂
k
a
o
c
g
k


52; 53.
48
44
8
9
40
39
13
17
35
31.30
22
26
re
sol
ut
fa
mi
la
ut
fa
mi
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mi
sol
ut
fa
re
la
mi
sol
fa
re
la
mi
fa
mi
ut
sol
re
la
fa
ut
sol


a+b.a.a+b
a+b.a.a+b.a

In y^e ~~Hyp~~parabola dm. suppose $ak=a=kk$

$ao=x$. $od=y$. $dc=$ ~~s~~ a secant. $ca=v$. $og=z$.

$xy=aa$. ~~yy~~ $yy=ss-xx+2vx-vv$

$-a^4+xxss-x^4+2vx^3=0$ Double roots equall

$+2 \quad \begin{matrix}-vvxx\\ 0\end{matrix} \quad -2 \quad -1 \;\Big|\; \dfrac{x^4-a^4}{x^3}=v.$

$x-v=\dfrac{a^4}{x^3}=oc$. $od:oc::kk:og$.

$\dfrac{aa}{x}:\dfrac{a^4}{x^3}::a:z$. $\dfrac{aaz}{x}=\dfrac{a^5}{x^3}$.

$zxx=a^3$. w^{ch} equation continues y^e nature of y^e crooked line gh. Now supposing y^e line og always moves over y^e same superficies in y^e same time, it will increase in ~~p~~ motion from kk in y^e same proportion y^t it decreaseth in lenght & y^e line oe will move uniformly ~~through~~ from (mq) soe y^t y^e space $mqen=gokk$. Suppose $ok=a$. $ao=2a$. $od=\frac{a}{2}=nm$. &

$mqen=\frac{1}{2}aa=ogkk$.

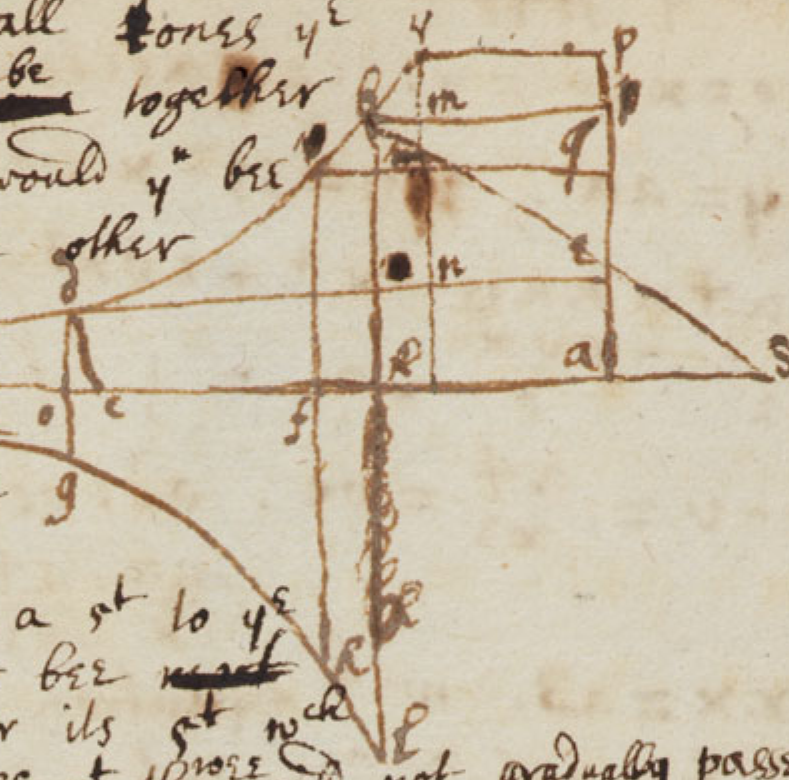
Modi

1	2	1	2	1	1	2	1	2	1	1	2	1
3	4	3	4	3	2	3	3	4	3	2	3	3
4	5	5	6	5	4	5	4	5	5	4	5	4
6	7	6	7	7	6	7	6	7	6	6	7	6
8	9	8	9	8	8	9	8	9	8	7	8	8
9	10	[illegible]	11	10	9	10	10	11	10	9	10	9
11	[illegible]	10	12	12	11	12	11	12	12	11	12	11
	12	11										

1. 6.

In y^{e} order of y^{e} musicall ~~tones~~ tones y^{e}
2 halfe notes may not be together
1st because every note would y^{n} bee
distant 3 tones from some other
w^{ch} is most ungratefull

2^{dly} whole notes ought to
bee interposed to moderate
their harshnesse.
3^{dly} since ~~their~~ must bee a 5^{t} to y^{e}
~~ground~~: these $\frac{1}{2}$ notes must bee ~~next~~
either next y^{e} ground or its 5^{t} w^{ch}
would make y^{m} harshe & y^{t} ~~they~~ notes could not gradually passe to
or from them.



Neither ought they to be distant
but one tone for y^{e} 2^{d} reason afforsd
& because they will bee more consonant
by y^{e} absense of more 3^{d} longs &c

if they be distant 2 tones
yet perhaps they may not be wholly
uselesse see a y^{e} last modes.

A Catalogue of y^{e} 12 Musicall modes in theire
order of gratefulnesse.

1	g . a . b c . d . e f . g	~~0.9~~ 9.17.22.31.39.44.53.
3	c . d . e f . g . a . b c	0.9.17.22.31.40.48.53.
2	d . e f . g . a . b c . d	0.9.14.22.31.40.45.53.
4	a . b c . d . e f . g . a	0.9.14.22.31.36.45.53
5	e f . g . a . b c . d . e	0.5.14.22.31.36.45.53.
6	f . g . a . b c . d . e f	0 9 17.26.31.40.48.53
	b c . d . e f . g . a . b	
	. b c . d . e f . g . a .	
	. a . b c . d . e f . g .	
	. d . e f . g . a . b c .	
	. g . a . b c . d . e f .	
	. e f . g . a . b c . d .	
	0.1.2.3.4.5.6.7.8.9.10.11.12	
2	g . a . b c . d e . f . g	g.a.b.cde.f.g
1	c . d e . f . g . a . b c	de.f.g.a.b.cd
3	d e . f . g . a . b c . d	f.g.a.bc.de.f.

suppose y^e line last found to be md. $mk=kh=a=ks$ |||

$ao=x$. $od=y$. $dc=s$. $ca=v$. $yxx=a^3$.

$yy=ss-vv+2vx-xx$. $\begin{array}{l} ssx^4+2vx^5-x^6-a^6=0 \\ -vv \\ \;\;0 \qquad\quad -1 \qquad -2 \;\; +4 \end{array}$

$v=\dfrac{x^6-2a^6}{x^5}$. $x-v=\dfrac{2a^6}{x^5}$. to find at

w^t point $do=oc$: $\dfrac{a^3}{xx}=\dfrac{2a^6}{x^5}$. $x^3=2a^3$.

$x=a\sqrt{c:2}=af$. $mq=fk=a$. $og=z$. $od:oc::fk:og$.

$\dfrac{a^3}{xx}:\dfrac{2a^6}{x^5}::a:z$. $a^3zx^5=2a^7xx$. $zx^3=2a^4$.

w^ch shews y^e nature of y^e line (gk). & $mneq=gofk$

or $nbpe=gokl$. suppose $ko=ka=a$. $oa=2a=x$

$od=\dfrac{a^3}{xx}=\dfrac{a^3}{4aa}=\dfrac{a}{4}$. $bn=\dfrac{3a}{4}$. $bpen=\dfrac{3aa}{4}=gokl$.

~~Suppose~~ $kl=ka=a=kb$. ~~$af=x=a\sqrt{c:2}$~~

~~$x-v=\dfrac{2a^6}{x^5}=\dfrac{2a^6}{a^5\sqrt{c:32}}=\dfrac{2a}{\sqrt{c:32}}=\dfrac{a}{\sqrt{c:4}}$.~~

~~$\dfrac{a^3}{aa}$~~

Suppose $kl=ka=kb=a$

$ak=x=a$. $bk=y=a$. $bs=s$. $as=v$.

$yy=ss-vv-2vx-xx$. $yyxxxx=a^6$.

$\begin{array}{l} ssx^4-2vx^5-x^6-a^6. \\ -vv \qquad -1 \qquad -2 \;\; +4 \\ \;\;0 \end{array}$ $\dfrac{-2x^6+4a^6}{2x^5}=v$

$v+x=\dfrac{2a^6}{x^5}=ks=2a$. $ks:bk::kl:fk$.

$2a:a::a:\dfrac{a}{2}$. $fk=\dfrac{a}{2}=ne=mq=rp$. $mn=\dfrac{3a}{4}=qe$

$\dfrac{3aa}{8}=mneq=lkog$. $ao=2a=x$. $\dfrac{a^3}{4aa}=do$

$do=\dfrac{a}{4}$. $oc=\dfrac{2a^6}{32a^5}=\dfrac{a}{16}$. $\dfrac{a}{4}:\dfrac{a}{16}::\dfrac{a}{2}:og=\dfrac{a}{8}$

$\dfrac{2a^6}{x^5}::\dfrac{a^3}{xx}:z::\dfrac{a}{2}$. $a^4=zx^3$.

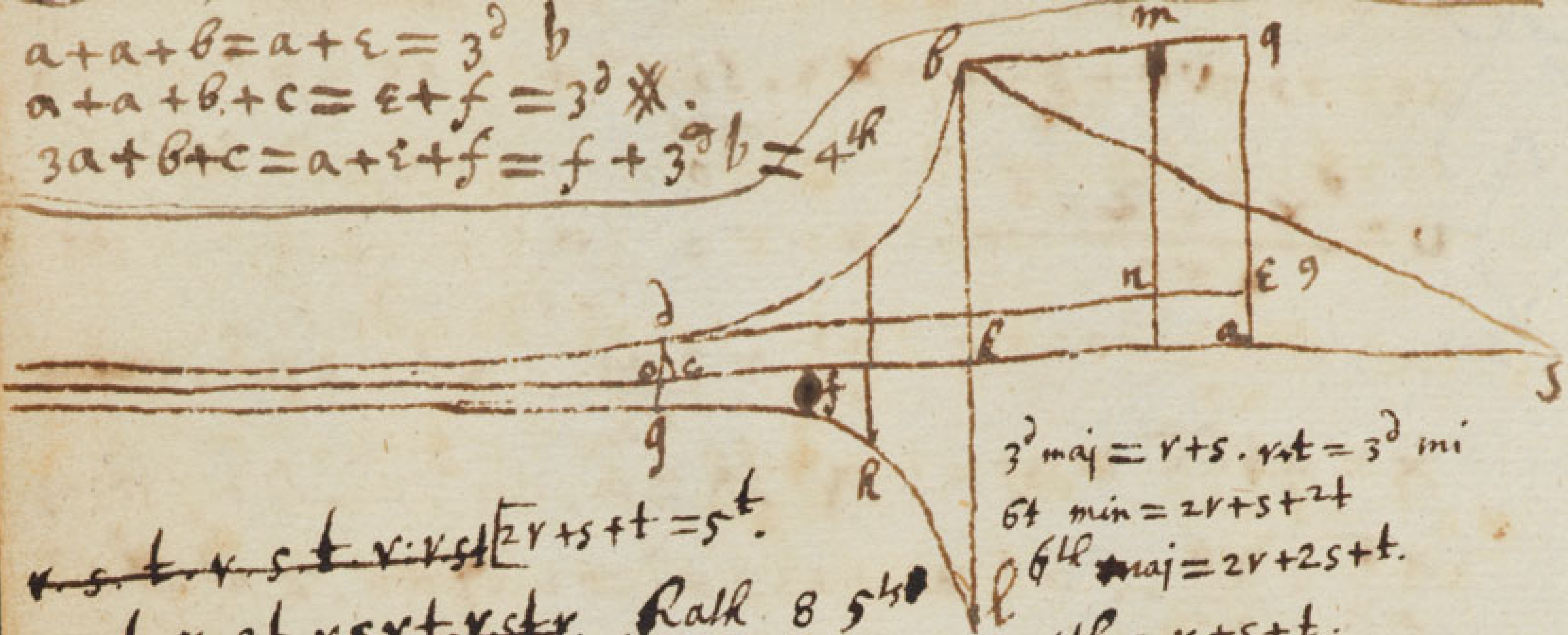
a, b, c = $\frac{1}{2}$ tone max; medi: minim:

~~a+b=d, a+c=e~~ a+a=d, a+b=e, a+c=f tone ma; me: mi

a+a+b = a+e = 3d ♭

a+a+b+c = e+f = 3d ✳.

3a+b+c = a+e+f = f + 3d ♭ = 4th



3d maj = r+s. r+t = 3d mi

6t min = 2r+s+2t

6th maj = 2r+2s+t.

4th = r+s+t.

~~r.s.t.r.s.t.r.r.s.t~~ 2r+s+t = 5t.

~~s.r.t.r.s.t.r.s.r.t.r.s.t.r.~~ Rath 8 5ths

~~1 2 3 4 5 6 7 8 5ts~~

~~1 2 3 4 5~~

r. s. t. r. s. t. r, r s t r s t r.
1 2 3 4 5 ~~6~~ ~~eights~~ fifts
1 0 2 third maj' 2
1 2 third min.
1 2. ~~third mind~~ 6ts maj
1 1 2 sixt minors
1 2 3 4 5 6 forths

r r t s r t s, r r t s
1 2 3 4 5 fourths 3
1 2 third maj.
1 2 third min.

6t mode r s r t s r t, r s
1 2 3 4 5. 4ths
1 2 3 3d maj
1. 2.3. 3d min

s r r t r s t, s

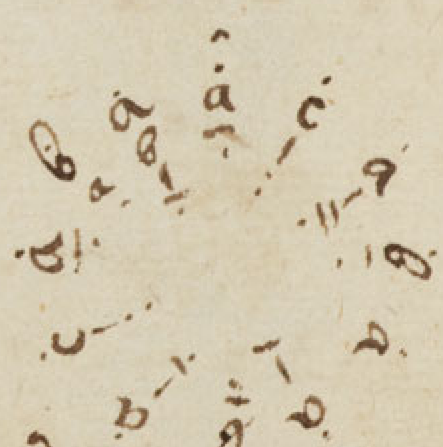
s r t r s t r, s. r. t. r s t r
1 2 3 4 5 fourths 3d mode
1 2 3 4 5 fifts
1 2 3 | 3d maj. 1
1 2 3 third mind
1 2 3 Sixt maj.
1 2 3 Sixt min.

ac, ab, a, ab, ac, a, ab; ac, ab
ba
ac, ba, aaa

abacaabacaba. a
. . b . c . b . c . b
abaacabaacab, a a
abacabaacaba

r s t r r t s, r s t r r ts.
1 2 3 4 fourths 4
1 2 third maj.
1 2 third minor

s r t r r t s, s r t.
1 2 fourths 5
1 3d maj
1 2 3 3d min.



9×4ths. 7×3ds ✳. 6 3ds ♭

v. r. t s s t r, r r t s
1 2 fourths 6
0 3d maj
1 2 3d min.

Suppose againe y^e last line whose nature is compris'd in this equation $x^3 = a^4$. $ak = bk = lk = a$

$ao = x$. $ac = v$. $do = y$. $dc = s$. $og = z$.

$$\begin{array}{ccccc} ssx^6 & +2vx^7 & -x^8 & -a^8 & =0 \\ -vv & -1 & -2 & +6 & \end{array} \qquad v = \frac{2x^8 - 6a^8}{2x^7}$$

$x - v = \frac{3a^8}{x^7}$. to find where $do = dc$

$\frac{a^4}{x^3} = \frac{3a^8}{x^7}$. $a^4x^4 = 3a^8$. $x^4 = 3a^4$. $x = a\sqrt{qq:3}$

$af = a\sqrt{qq:3}$ | $bk = y = a$. $as = s$. $as = v$. $ak = x = a$ | $x + v = \frac{3a^8}{a^7} =$

$x + v = 3a$. $3a : a :: kl(=a) : fh = \frac{a}{3} = mq = ne$

$oa = x = 2a$. $\frac{a^4}{x^3} = do = y = \frac{a^4}{8a^3} = \frac{a}{8} = do$.

$mn = \frac{7a}{8} = qe$. $mqen = \frac{7aa}{24} = lkog = \frac{7aa}{24}$.

rst rst r; rs 2x3d b. 2x3d ✱. 5x4th. 1st mode 2d mode. 3d no 4th	srt rst r; sr 3x3d b. 3x3d ✱. 5x4th. 6t, 4th, & 5t mode 3d mode. 1st mode. no 2d	rst srt r, rs 2x3d b. 2x3d ✱. 3x4th.	srt srt r, sr 3x3d b. 3x3d ✱ ~~5x4th~~ 5t, 4th mod. 3d mode no 1st 6t mode. 2d mode
rrt sst r; rr 2x3d b. 0x3d ✱ 2x4th.	sst rrt r; ss 3x3d b. 1x3d ✱. 2x4th	rst rrt s; rs 2x3d b. 2x3d ✱ 4x4th. 2d mod.	srt rrt s; sr. 3x3d b. 1x3d ✱ 2x4ths.
rrt rst s; rr. 2x3d b. 2x3d ✱. 3x5t.	rrt srt s, rr 2x3d b. 2x3d ✱. 5x4ths 2d. no a 3d 4th mode. 6t mode		

1. e	f	g	a	b	c	d	e		5
2. a	b	c	d	e	f	g	a		4
3. d	e	f	g	a	b	c	d		2
4. g	a	b	c	d	e	f	g		1
5. c	d	e	f	g	a	b	c		3
6. f	g	a	b	c	d	e	f		6

r = ton. maj. s = ton min. t = semit maj. v = semit min.

1st. gd = cg = da = 5t = 2r+s+t. ~~gf~~ dg = gc = ad = r+s+t.
cd = ~~ga~~ = r. ac = s+t. ca = 3r+s+t. ab = s. be = t.
dc = 2r+2s+2t. $\boxed{\text{de = ~~rs~~ s. ef = t. per sup. et fg}}$

3 | s t r r s t r b Modus harum ~~optimus~~ vocum respectu fundamenti.
4 | r s t r s t r
5 | r s t r r s t

2d. gd = da = ae = 5ta = 2r+s+t. dg = ad = ea = r+s+t. ga = da − dg =
de = ae − ad = r. ge = 3r+s+t = gd+de. eg = s+t. ef = t. fg = s.
And if ab = s. be = t. then cd = r & y^e voyces in respect of their ~~base~~ grand are best.

2 | st rrts v.b
3 | rts vst r.
4 | rst rrt s.

3d. fc = cg = gd = 2r+s+t. cf = gc = dg = r+s+t. fg = cg − cf = cd = gd − gc = r.
df = cf − cd = s+t. de = s. ef = t. & if ga = s, y^n ab = r. be = t
whence 4. srtrstr b
5. rstrsrt
6. rsrtrst
Or if ga = t. y^n ab = s & be = t whence
4 rstrstr.
5 rstrrst.
6 rrstrst.
These differ in y^t this hath but 2 exact 2ds & this but 2 exact thirds.

If in y^e 1st case de = r. y^n ab = r then
2. rtsrtsr.b
3. rtsrrts.
4 rrtsrts.

3. rtsrstr.b
4. rstrrts.
5. rrtsrst.
If in y^e 2d case

2. s t r r t s r.	1. t s r r t s r.	3. s t r r s t r.
3 r t s r s t r.	2. r t s r t s r.	4. r s t r s t r.
4 r s t r r t s.	3. r t s r r t s.	5. r s t r r s t.
5 r r t s r s t.	4. r r t s r t s.	6. r r s t r s t.

Likewise supposing y^e line $yx^4 = a^5$.

$x - v = \frac{4a^{10}}{x^9} = oc$. $af = \sqrt[9]{qc:4}$. $ka\,(=x) + v = 4a$

$fk = \frac{a}{4}$. $do = \frac{a}{16}$, $mq = ne = \frac{a}{4} . \frac{15a}{16} \Big| \frac{15aa}{64}$. &c

whence supposing x to be a line increasing in arithmeticall proportion from y^e quantity of y^e line (a) untill it be as long as b. y^e superficies resulting out of $\frac{a^3}{xx}$. $\frac{a^4}{x^3}$ &c is found as follows.

$\frac{a^3}{xx} = aa - \frac{a^3}{b}$. $\frac{a^6}{x^5} = \frac{aa}{4} - \frac{a^6}{4b^4}$

$\frac{a^4}{x^3} = \frac{aa}{2} - \frac{a^4}{2bb}$. $\frac{a^7}{x^6} = \frac{aa}{5} - \frac{a^7}{5b^5}$.

$\frac{a^5}{x^4} = \frac{aa}{3} - \frac{a^5}{3b^3}$. $\frac{a^8}{x^7} = \frac{aa}{6} - \frac{a^8}{6b^6}$. &c

1	trsrtsr
2	rtsrtrs
3	rtrsrts

1 Of y^e Key or Ground sound. 2^{dly} Of its 8^{ths}. 3^{dly}, of their divisions into 5^{ts} & 4^{ths} 6^{ts} & 3^{ds}, illustrated by y^e division of a corde. 4^{thly}, The Order of y^e concords in respect of gratefulnes deduced thence & from other considerations. 5^{thly} y^e degrees deduced thence & of their proportion of y^e concords & degrees i.e. y^e logarithmes of their strings. 6 Of y^e various ordering of y^e degrees, & distance of y^e halfe notes, y^e Keys fift being onely stable 7 Of y^e moodes arising thence & their dignity, explained by one line, o.p. qr.s.tv.o.p.qr.s.tv.o.p. &c. 8^{thly}, How y^e tones major & minor are best ordered in every Moode.. 9^{thly} of passing from one moode to another explained by 3 lines g.a.bc.d.ef.g
c.d.ef.g.a.bc
f.g.a.bc.d.ef

10 How y^e notes major & minor to be ordered for y^e purpose.

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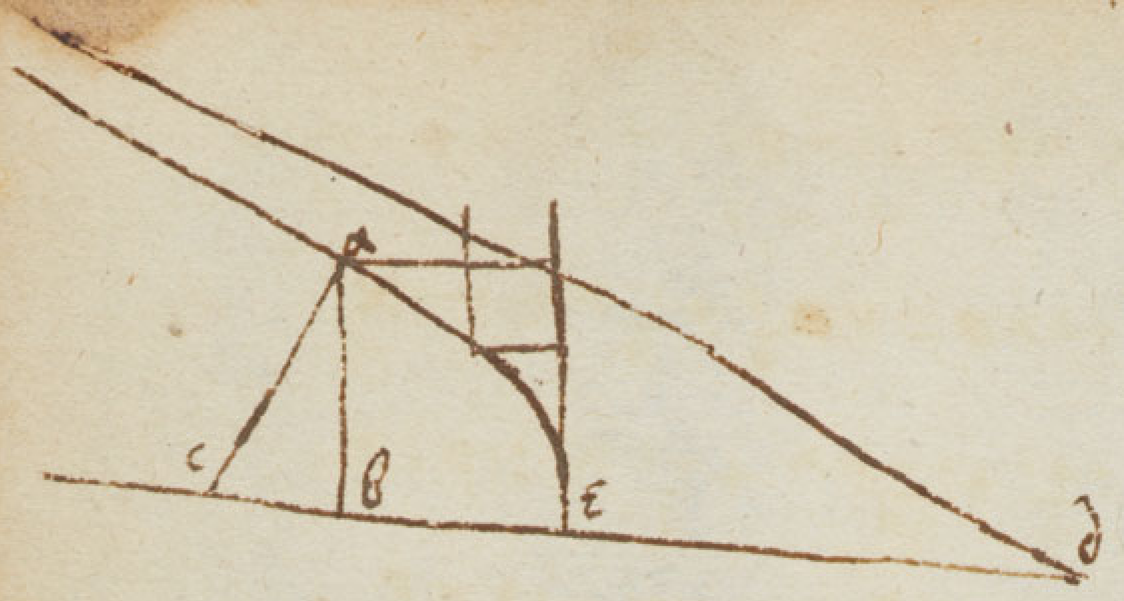

a
c
b
e
d

in y^e ~~Hyp~~ Hyperb: $\varepsilon\delta=q$. $b\varepsilon=x$ $ab=y$. 115

$$\frac{r}{1}x+\frac{r}{q}xx=yy=ss-xx^2+2vx-vv$$

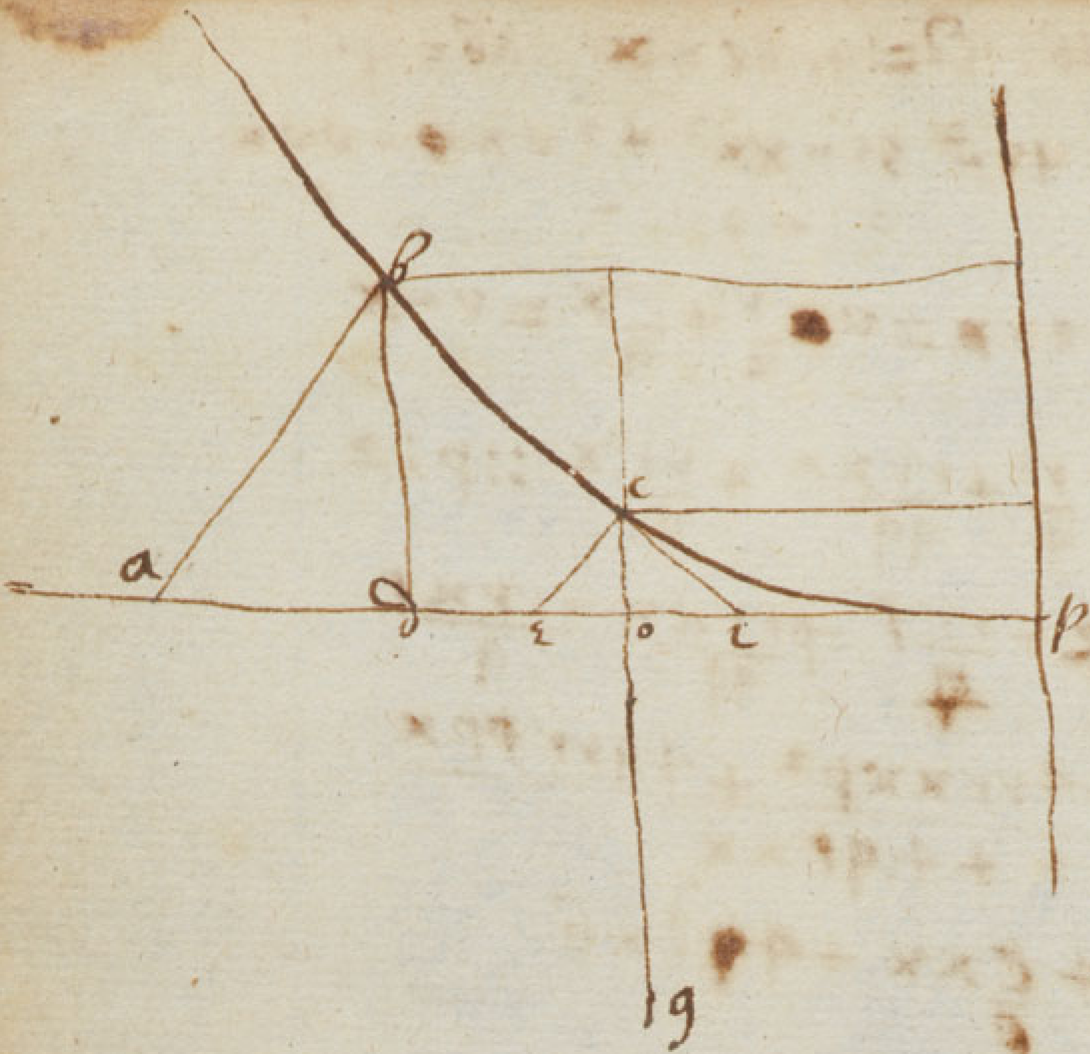
$$\frac{r}{2}+\frac{r}{q}x=v-x$$

$$rx+\frac{r}{q}xx:\frac{rr}{4}+\frac{rrxx}{qq}+\frac{rrx}{q}::p:z.$$

$$rxz+\frac{r}{q}xxz-\frac{rrp}{4}-\frac{prrxx}{qq}-\frac{rrxp}{q}$$

$$zz=\frac{p^2qqrr+4rrxxp^2+4qrrppx}{4qqrx+4qrxx}$$

$$zz=\frac{bbb+4\frac{\varepsilon}{\delta}xx+4q)x\,b}{qx+xx}$$



b
c
a
d
ε
o
i
p
g

$pd = x.\ db = y.\ ayy = x^3.\quad ap = v.\ ab = s.\ op = og = b.$

$\underset{0}{ass} - \underset{0}{avv} + \underset{1}{2av\cancel{x}} - \underset{2}{axx} - \underset{3}{x^3} = 0$

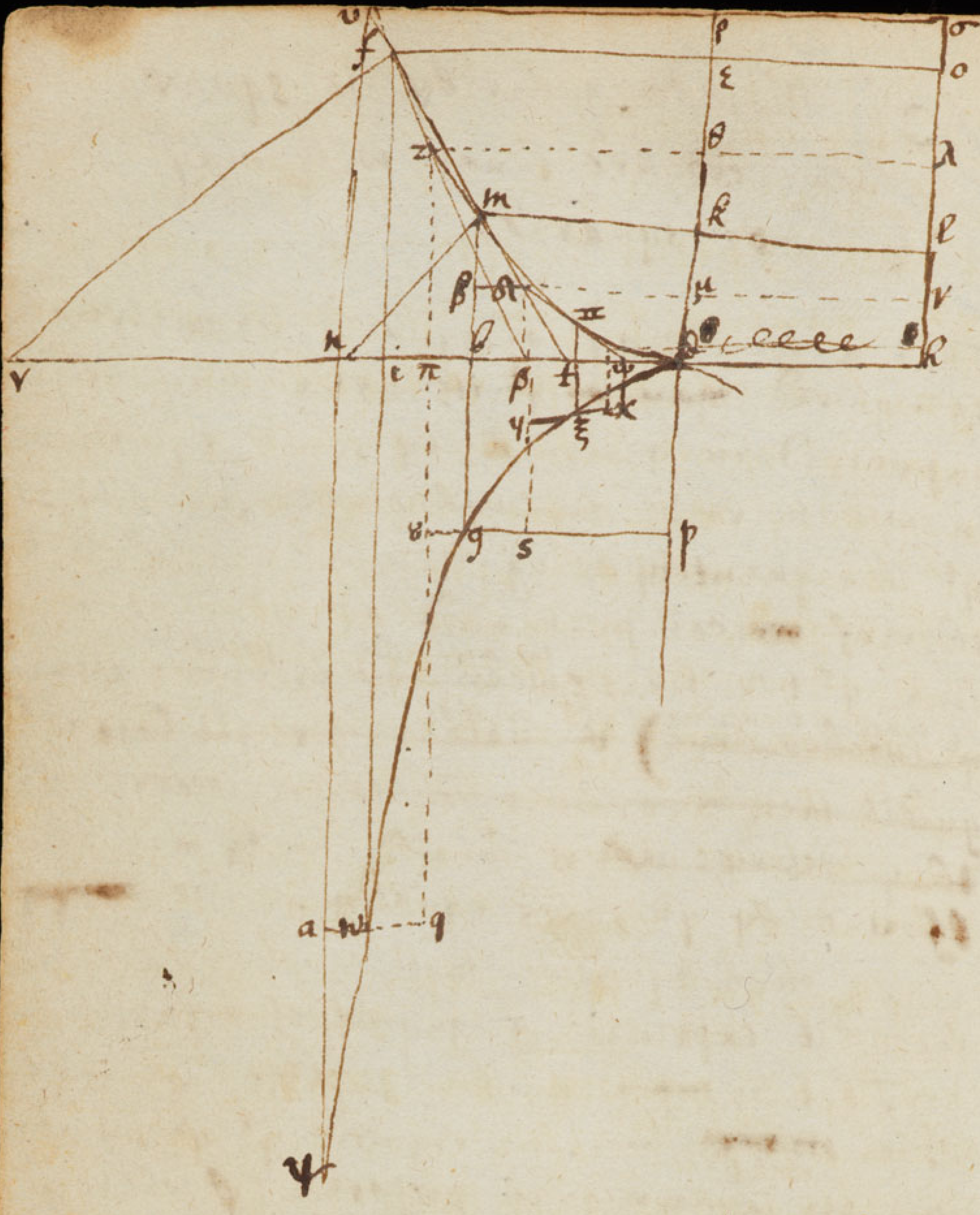
$\dfrac{{}^2axx + {}^3x^3}{2ax} = v \quad v - x = \dfrac{3xx}{2a} = ad : ad^2 = \dfrac{9x^4}{4aa}.$

$\dfrac{x^3}{a} : \dfrac{9x^4}{4a^2} :: bb : zz.\quad \dfrac{z^2x^3}{a} = \dfrac{9x^4b}{4aa}.\quad 4az^2 = 9b^2x.$

A Method whereby to square those crooked lines wch may be squared.

That a line may be squared Geometrically tis required ~~may be~~ yt its area may be expressed in generall by some equation in wch there is an unknowne quantity, so yt this quantity being determined ye area thereof ~~[illegible]~~ (comprehended by ye crooked line, ye two lines ~~(whose intersections describe ye crooked line)~~ to wch all ye points in ye crooked line are referred), ~~& another streight line wch guides these two lines in theire motion wn they describe it~~) is limited ~~& [illegible]~~ may bee found by ye same equation. Also ~~[illegible]~~ every such equation must be of two dimensions because it expresseth ye quantity of a superficies.

That an ~~super~~ equation expresse ye area of a ~~straight~~ crooked line tis required yt ye superficies increase in an ~~[illegible]~~ unequall proportion, ~~&~~ when ye line (considered as unknowne) increaseth ~~[illegible]~~ in arithmeticall proportion, wherefore (supposing x always to signifie ye unknowne quantity: a, b, c, &c; to signifie ye quantitys given) ax, or xx either alone or added to any other supperficies, serve not to find ye area of any crooked line wch may not be found wth out ym.



f
σ
o
ε
θ
λ
m
k
l
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ν
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K
γ
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π
β
x
ξ
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s
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q
ψ

Prop:

Haveing an equation of 2 dimensions to find wt crooke line it is ~~of~~ whose area it doth expresse. suppose ye equation is $\frac{x^3}{a}$. nameing ye quantitys; $a = dk = kl$. ~~...~~ $bg = y$. $db = mk = x = gp$. ye superficies $dbg = \frac{x^3}{a}$ supose ye square $dkhl$ ~~...~~ is equall to ye superficies gbd; ye ~~dk~~ $z = bm = lk = \frac{x^3}{aa}$, & $aaz = x^3$. wch is an equation expressing ye nature of ye line fmd.

a line wch cutteth ~~f~~ dmf at right angles

Next makeing $nm = s$, $nd = v$.

$ss - vv + 2vx - xx = \frac{x^6}{a^4} =$ mb squared. wch is an equation haveing 2 equall rootes & therefore multiplyed according to Huddenius his method, produceth another.

0 0 1. 2. 6.

$2vx = 2xx + \frac{6x^6}{a^4}$

$v = x + \frac{3x^5}{a^4}$. &

$nb = v - x = \frac{3x^5}{a^4}$. Now supposeing, $mb : bn :: dk : bg$.

that is, $3\frac{x^5}{a^4} : \frac{x^3}{aa} :: y : a$. $\left\{ \begin{array}{l} \frac{3xx}{a} = ay. \\ 3xx = a^2y. \end{array} \right.$

Which is ye nature of ye line dgw & ye area $dbg = dklh = \frac{x^3}{a}$, makeing $db = x$. $dk = a$. Or $diw = deoh = \frac{x^3}{a}$, determining (di) to be (x). &c

The Demonstration whereof is as followeth

Suppose ~~wx~~ $\omega\delta$, δmz, zfv &c are tangents of ye line dmf. & from theire intersections z, δ, v, draw va, zq. $\delta\varsigma$. ωx. ~~parallel~~ & from theire touch points draw fw, mg, ~~...~~. all parallell to ~~...~~ kp. also from ye same point of intersection draw vσ, ~~...~~ zλ, δρψ. ωξ.

~~And nb. bm :: bm : bt :: bg. bf. βm. βd.~~

And mb : nb :: bt : bm :: ~~δ~~ δβ : βm :: kl : bg.
wherefore $\delta\beta \times bg = \beta m \times kl$. that is
ye rectangle $kl\nu\mu = bpsg$. And.
$\pi\rho s\gamma = \theta\lambda\nu\mu$. in like manner it may
bee demonstrated yt $aq\pi\nu = \theta\lambda\sigma\rho$, &
$\rho\omega x y = \mu\delta\nu k$. ~~&~~ so yt ye rectangle
pskd is equall to any number of such
like squares inscribed twixt ye line ny
& ye point d, wch squares if they bee
infinite in number, they will bee
equall to ye superficies dnywgξ.

This being demonstrated that I may
shunne confusion in ye squareing ye
lines of every sort I shall use this metho[d]
~~to~~ in distinguishing ym. viz: first such lines
whose area is exprest by equations in wch
ye unknowne quantity is numerator, & yt 1st
all ye sines being affirmative, 2dly mixed.
2dly lines whose area is exprest by quantitys
in wch ye unknowne quantity is divisor, & those
1st under affirmative sines, 2d under mixt ones.
3 lines squared by equations mixt of ye 2 former
kinds, whose quantitys are all 1s affirmative
2dly mixt.

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The squareing of those lines whose area is exprest by affirmative quantitys in w^ch^ y^e^ unknowne quantity is numerat^r^

The equations expressing the nature of y^e^ lines. | Theire square.

$\cancel{x^3}$ $3xx = ay$. Parab: $\quad \dfrac{x^3}{a}$.

$4x^3 = aay$. ---- $\dfrac{x^4}{aa}$

$5x^4 = ya^3$ ---- $\dfrac{x^5}{a^3}$

$6x^5 = ya^4$ ---- $\dfrac{x^6}{a^4}$

$7x^6 = ya^5$ ---- $\dfrac{x^7}{a^5}$.

& soe infinitely.

~~$4x^3 + 7bx^3 + 3bb\,xx - bay - bbay = 0$. $\quad x^3 + x^4$.~~

~~$5x^6 + 8bbx^4 + 3b^4x^2 - ab^4y = 0$. $\quad \dfrac{x^3}{a} + \dfrac{x^5}{abb}$~~

~~$- abbyx^2$~~

$4x^3 + 3bx^2 = bay \quad \dfrac{x^3}{a} + \dfrac{x^4}{ab}$

$5x^4 + 3b^2x^2 = bbay \quad \dfrac{x^3}{a} + \dfrac{x^5}{abb}$

$6x^5 + 3b^3x^2 = b^3ay \quad \dfrac{x^3}{a} + \dfrac{x^6}{ab^3}$

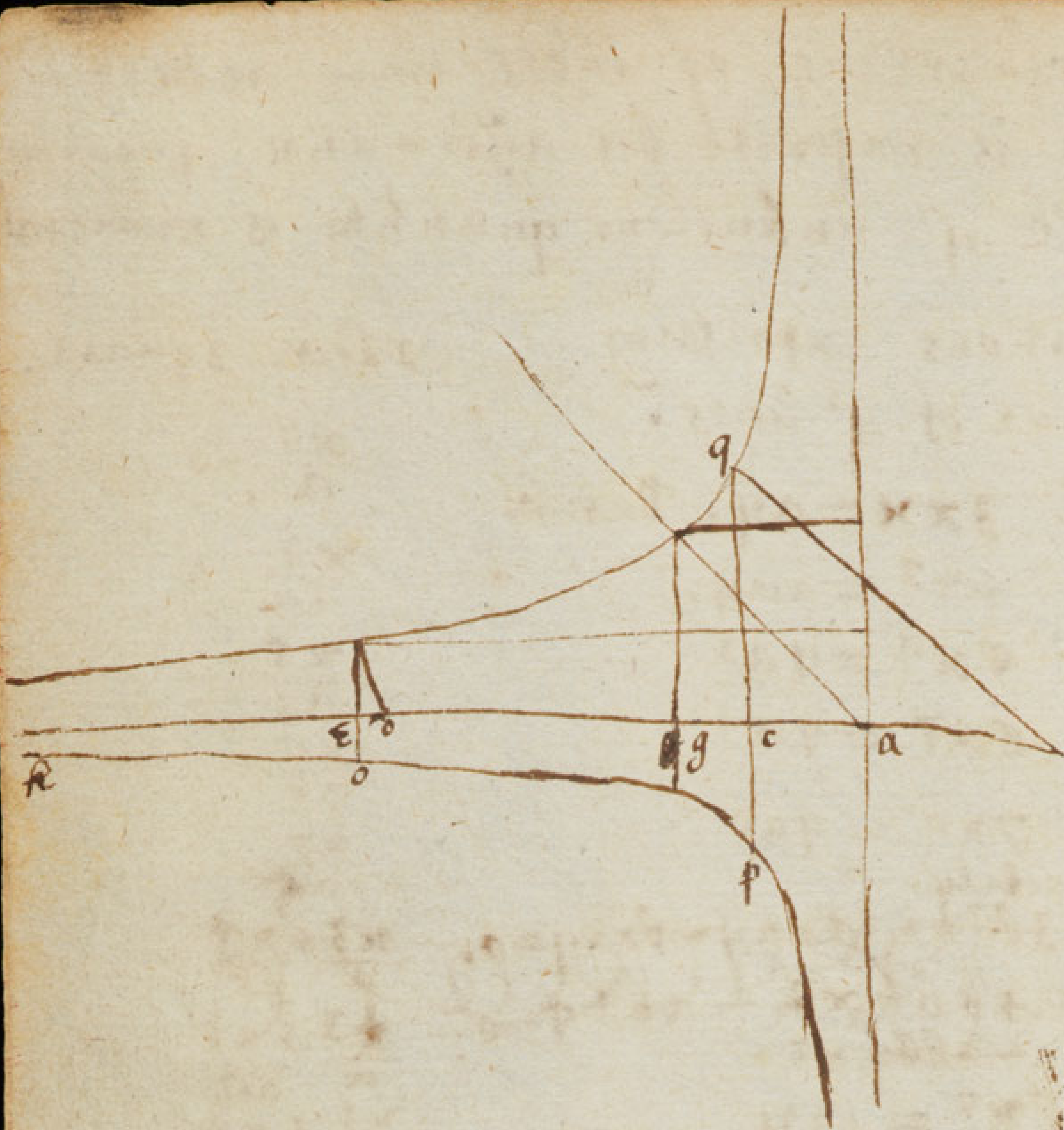
$7x^6 + 3b^4x^2 = b^4ay \quad \dfrac{x^3}{a} + \dfrac{x^7}{ab^4}$

Soe y^t^ y^e^ nature of every crooked line, whose area is compounded of y^e^ area of 2 or more of y^e^ former lines, or of y^e^ difference of y^e^ area of 2 or more of y^e^ former lines, is exprest by an equation compounded ~~of y^e^ areas~~ of y^e^ Equations expresing y^e^ nature of ~~th~~ose lines.

$4x^3 - 3bx^2 = bay$ ---- $\dfrac{x^4 - x^3b}{ab}$

$5x^4 - 3bbx^2 = bbay \quad \dfrac{x^5 - x^3bb}{abb}$

$6x^5 - 3b^3x^2 = b^3ay \quad \dfrac{x^6}{ab^3} - \dfrac{x^3}{a}$.


q
E
o
g
c
a
p

The squareing those lines whose area is exprest by an equation in wch ye unknown quantity is denominator. 125

The Equations expressing ye nature of ye line.	The square thereof when ~~x greater~~ x lesse then a
$xxy = a^3$	$\frac{a^3}{x}$
$x^3y = {}^2a^4$	$\frac{a^4}{xx}$
$x^4y = 3a^5$	$\frac{a^5}{x^3}$
$x^5y = 4a^6$	$\frac{a^6}{x^4}$
$x^6y = 5a^7$	$\frac{a^7}{x^5}$
$4yy = 9ax$ Parab.	$x\sqrt{ax}$
$4ayy = 25x^3$	$\frac{xx}{a}\sqrt{ax}$
$4a^3yy = 49x^5$	$\frac{x^3}{aa}\sqrt{ax}$
$4a^5yy = 81x^7$	$\frac{x^4}{a^3}\sqrt{ax}$
$4a^7yy = 121x^9$	$\frac{x^5}{a^4}\sqrt{ax}$
$4xyy = a^3$	$a\sqrt{ax}$
$4x^3y^2 = a^5$	$\frac{aa}{x}\sqrt{ax}$
$4x^5y^2 = 9a^7$	$\frac{a^3}{xx}\sqrt{ax}$
$4x^7y^2 = 25a^9$	$\frac{a^4}{x^3}\sqrt{ax}$
$4x^9y^2 = 49a^{11}$	$\frac{a^5}{x^4}\sqrt{ax}$

$9ax^2 + 12aax + 4a^3 = 4yyx + 4ayy.$ $\quad x\sqrt{ax+aa}$

$25x^4 + 40ax^3 + 16aax^2 = 4axy^2 + 4aay^2.$ $\quad \frac{x^2}{a}\sqrt{ax+aa}$

$49x^6 + 84ax^5 + 36a^2x^4 = 4a^3xy^2 + 4a^4y^2.$ $\quad \frac{x^3}{aa}\sqrt{ax+aa}$

$81x^8 + 144ax^7 + 64aax^6 = 4a^5xy^2 + 4a^6y^2.$ $\quad \frac{x^4}{a^3}\sqrt{ax+aa}$

$a^3 = 4xy^2 + 4ayy.$ $\quad a\sqrt{ax+aa}$

$a^5x^2 + 4a^6x + 4a^7 = 4x^5yy + 4ax^4y^2.$ $\quad \frac{aa}{x}\sqrt{ax+aa}$

$9a^7x^2 + 24a^8x + 16a^9 = 4x^7y^2 + 4ax^6y^2.$ $\quad \frac{a^3}{x^2}\sqrt{ax+aa}$

$25a^9x^2 + 60a^{10}x + 36a^{11} = 4x^9y^2 + 4ax^8y^2.$ $\quad \frac{a^4}{x^3}\sqrt{ax+aa}.$

$9ax^2 - 12aax + 4a^3 = 4xy^2 - 4ayy.$ $\quad x\sqrt{ax-aa}$

$25x^4 - 40ax^3 + 16aax^2 = 4axy^2 - 4aayy.$ $\quad \frac{x^2}{a}\sqrt{ax-aa}$

$a^3 = 4xy^2 - 4ay^2.$ $\quad a\sqrt{ax-aa}$

$a^5x^2 - 4a^6x + 4a^7 = 4x^5yy - 4ax^4yy.$ $\quad \frac{aa}{x}\sqrt{ax-aa}$

$9ax^2 - 12aax + 4a^3 = 4ayy - 4xyy.$ $\quad x\sqrt{aa-ax}$

$25x^4 - 40ax^3 + 16aax^2 = 4a^2yy - 4axyy.$ $\quad \frac{x^2}{a}\sqrt{aa-ax}$

$a^3 = 4ay^2 - 4xy^2.$ $\quad a\sqrt{aa-ax}$

$a^5x^2 - 4a^6x + 4a^7 = 4ax^4yy - 4x^5yy.$ $\quad \frac{aa}{x}\sqrt{aa-ax}.$

Note y^t the lines whose nature is exprest by y^e 4 latter sorts of equations, are y^e same w^th the lines of y^e 2 former sorts. Doubtfull.

$9aax + 24axx + 16x^3 = 4xyy + 4ayy.$ $\quad x\sqrt{ax+xx}$

$25aax^3 + 60ax^4 + 36x^5 = 4aaxyy + 4a^3yy.$ $\quad \frac{xx}{a}\sqrt{ax+xx}$

$49aax^5 + 112ax^6 + 64x^7 = 4a^4xyy + 4a^5yy =$ $\quad \frac{x^3}{aa}\sqrt{ax+xx}$

$81aax^7 + 180ax^8 + 100x^9 = 4a^6xyy + 4a^7yy =$ $\quad \frac{x^4}{a^3}\sqrt{ax+xx}$

$a^4 + 4a^3x + 4aaxx = 4axyy + 4xxyy.$ $\quad a\sqrt{ax+xx}.$

$a^6 = 4ax^3yy + 4x^4yy.$ $\quad \frac{aa}{x}\sqrt{ax+x^2}.$

$9a^8 + 12a^7x + 4a^6x^2 = 4a^4x^5yy + 4x^6y^2.$ $\quad \frac{a^3}{xx}\sqrt{ax+xx}.$

$25a^{10} + 40a^9x + 16a^8x^2 = 4a^6x^7yy + 4x^8yy.$ $\quad \frac{a^4}{xx}\sqrt{ax+xx}.$

$9aax - 24axx + 16x^3 = 4ayy - 4xyy.$ $\quad x\sqrt{ax-xx}.$

$x\sqrt{aa+xx}$

$4x^4 + 4aaxx + a^4 = xxyy + aayy$

$9x^6 + 12aax^4 + 4a^4x^2 = aaxxy^2 + a^4y^2.$ $\quad \frac{xx}{a}\sqrt{aa+xx}$

$16x^8 + 24aax^6 + 9a^4x^4 = a^4x^2y^2 + a^6y^2.$ $\quad \frac{x^3}{a^2}\sqrt{aa+xx}$

$25x^{10} + 40aax^8 + 16a^4x^6 = a^6x^2y^2 + a^8y^2.$ $\quad \frac{x^4}{a^3}\sqrt{aa+xx}.$

$a\sqrt{aa+xx}$

$aaxx = aayy + xxyy.$

$\frac{aa}{x}\sqrt{aa+xx}$

$a^8 = aax^4yy + x^6yy$

$4a^{10} + 4a^8xx + a^6x^4 = aax^6yy + x^8yy.$ $\quad \frac{a^3}{xx}\sqrt{aa+xx}$

$9a^{12} + 12a^{10}xx + 4a^8x^4 = aax^8y^2 + x^{10}yy.$ $\quad \frac{a^4}{x^3}\sqrt{aa+xx}.$

$4x^4 - 4aaxx + a^4 = aayy - xxyy.$ $\quad x\sqrt{aa-xx}$

$x\sqrt{aa-xx}.$

$aaxx = aayy - xxyy.$

$\frac{aa\sqrt{aa-xx}}{x}$

$a^8 = aax^4yy - x^6yy$

$a\sqrt{\frac{a^3-axx}{x}}.$

$a^4 + {}^3a^2bx + {}^4aax^2 + \frac{9}{4}bbx^2 + {}^6bx^3 + 4x^4 = \begin{array}{l} a^2y^2. \\ +bx \\ +xx \end{array}$ $\quad x\sqrt{aa+bx+xx}.$

$aabb + 4aabx + 4aaxx = 4ccyy + {}^4bxy^2 + {}^4xxy^2. \quad a\sqrt{cc+bx+xx}$

$\sqrt{a^3x + x^4}$

$\sqrt{a^4 + ax^3}$

$9ax^4 + {}^6a^3x^2 + a^5 = 4aaxyy + {}^4x^3yy. \quad \sqrt{a^3x + ax^3}.$

$\sqrt{a^4 + x^4}.$

$\frac{a^3}{a+x}$

$\frac{a^2 x}{a+x}$

$\frac{a x^2}{a+x}$

$\frac{x^3}{a+x}$

$\frac{a^3}{a-x}$

$\frac{a^3}{x-a}$

$\cancel{x^4 + 3ax^3 + 3ax^3 + 3a^2x^2 + a^3x \quad a^4 = a^3y + 2a^2xy + ax^2y.}$

$a^3 = bby + 2bxy + xxy.$ $\frac{a^3}{b+x}$

$2b^4x + b^4a = x^4y + 2ax^3y + aaxxy$ $\frac{b^4}{ax+x^2}$

$3b^5x + 2b^5a = x^5y + 2ax^4y + a^2x^3y$ $\frac{b^5}{ax^2+x^3}$

$4b^6x + 3b^6a = x^6y + 2ax^5y + a^2x^4y$ $\frac{b^6}{ax^3+x^4}$

$\frac{x^3}{a+x}$

$\frac{x^4}{aa+ax}$

$\frac{x^5}{a^3+xa^2}$

$\frac{x^6}{a^4+xa^3}$

$aab = bby + 2bxy + xxy.$ $\frac{a^2x}{b+x}$

$2bax + axx = bby + 2bxy + xxy.$ $\frac{ax^2}{b+x}$

$2a^4x = b^4y + 2bbxxy + x^4y$

$$\frac{a^4}{bb+x^2}$$

$$\frac{a^5}{bbx+x^3}$$

$$\frac{a^6}{bbx^2+x^4}$$

$$\frac{a^7}{bbx^3+x^5}$$

$$\frac{x^4}{bb+x^2}$$

$$\frac{x^5}{b^3+x^2b}$$

$$\frac{x^6}{b^4+bbx^2}$$

$$\frac{x^7}{b^5+bbx^2}$$

$$\frac{a^3x}{bb+x^2}$$

$$\frac{aaxx}{bb+xx}$$

$$\frac{ax^3}{bb+xx}$$

$27x^2y^3 = a^5$ —— $a\sqrt{c:aax}$.

$27x^5y^3 = 8a^8$ —— $\frac{aa\sqrt{c:aax}}{x}$

$27x^8y^3 = 125a^{11}$ —— $\frac{a^3}{x^2}\sqrt{c:aax}$

$27x^{11}y^3 = 512a^{14}$ —— $\frac{a^4}{x^3}\sqrt{c:aax}$

$27xy^3 = 8a^4$. —— $a\sqrt{c:axx}$.

$27x^4y^3 = a^7$ —— $\frac{aa}{x}\sqrt{c:axx}$.

$27x^7y^3 = 64a^{10}$ —— $\frac{a^3}{xx}\sqrt{c:axx}$.

$27x^{10}y^3 = 343a^{13}$ —— $\frac{a^4}{x^3}\sqrt{c:axx}$.

$\cancel{27y^3 = 64aax}$

$\cancel{27ay^3 = 343x^4}$

$27y^3 = 64aax$ —— $x\sqrt{c:aax}$.

$27ay^3 = 343x^4$ —— $\frac{xx}{a}\sqrt{c:aax}$.

$27a^4y^3 = 1000x^7$ —— $\frac{x^3}{aa}\sqrt{c:aax}$.

$27y^3 = 125ax^2$ —— $x\sqrt{c:axx}$

$27a^2y^3 = 512x^5$ —— $\frac{xx}{a}\sqrt{c:axx}$

$27a^5y^3 = 1331x^8$ —— $\frac{x^3}{a}\sqrt{c:axx}$.

$$\frac{a^5}{b^3+x^3}$$

$$\frac{a^6}{b^3x+x^4}$$

$$\frac{a^7}{b^3x^2+x^5}$$

$$\frac{a^8}{b^3x^3+x^6}$$

$$\frac{x^5}{b^3+x^3}$$

$$\frac{x^6}{b^4+x^3b}$$

$$\frac{x^7}{b^5+bbx^3}$$

$$\frac{x^8}{b^6+b^3x^3}$$

$$\frac{a^4x}{b^3+x^3}$$

$$\frac{a^3x^2}{b^3+x^3}$$

$$\frac{a^2x^3}{b^3+x^3}$$

$$\frac{ax^4}{b^3+x^3}$$

$256\,x^3y^4 = a^7.$ ——— $a\sqrt{qq:aaax}.$

$256\,x^7y^4 = 81\,a^{11}.$ ——— $\frac{aa}{x}\sqrt{qq:aaax}.$

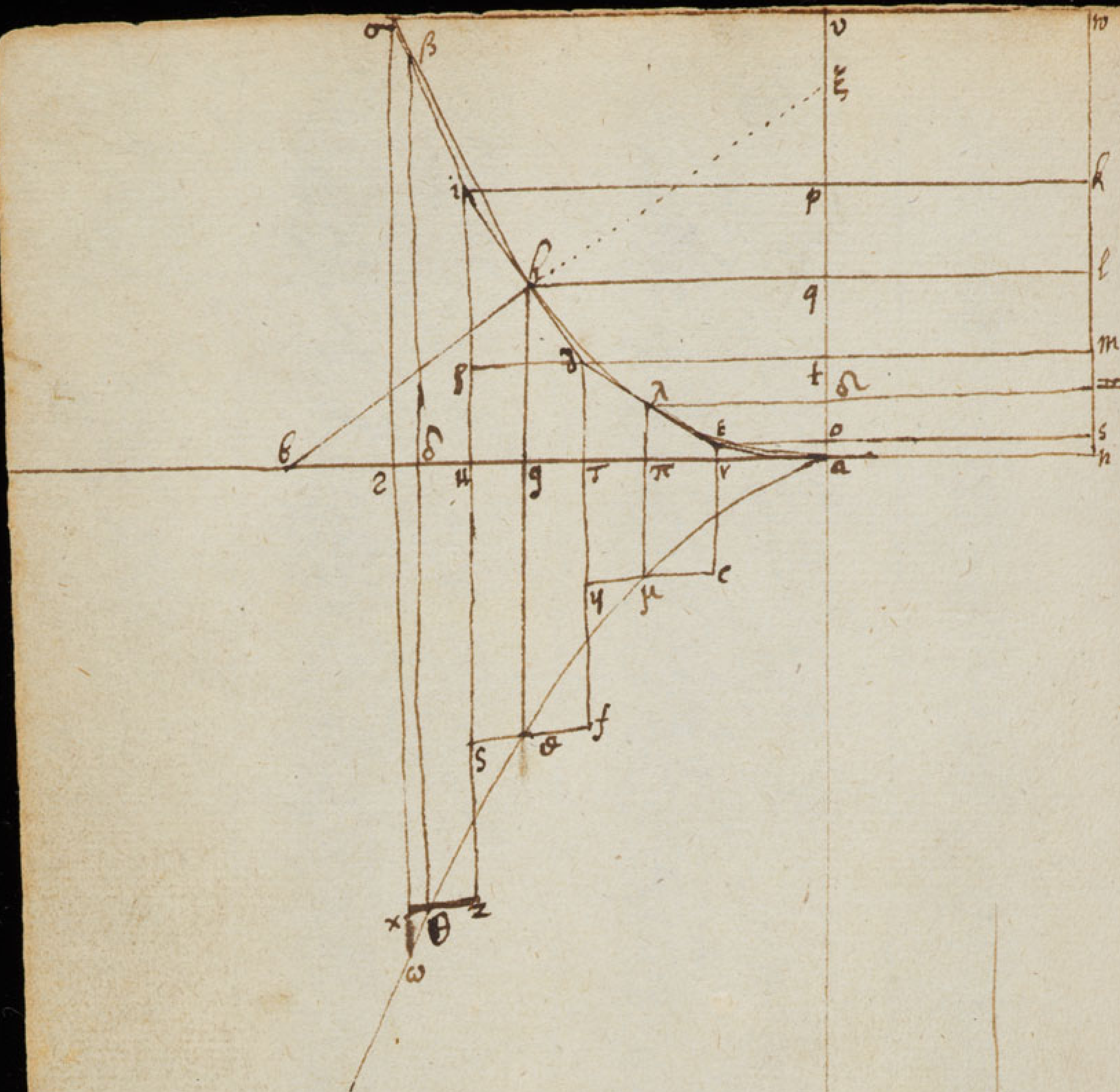
$256\,x^{11}y^4 = 2401\,a^{15}$ ——— $aaa\sqrt{qq:aaax}.$

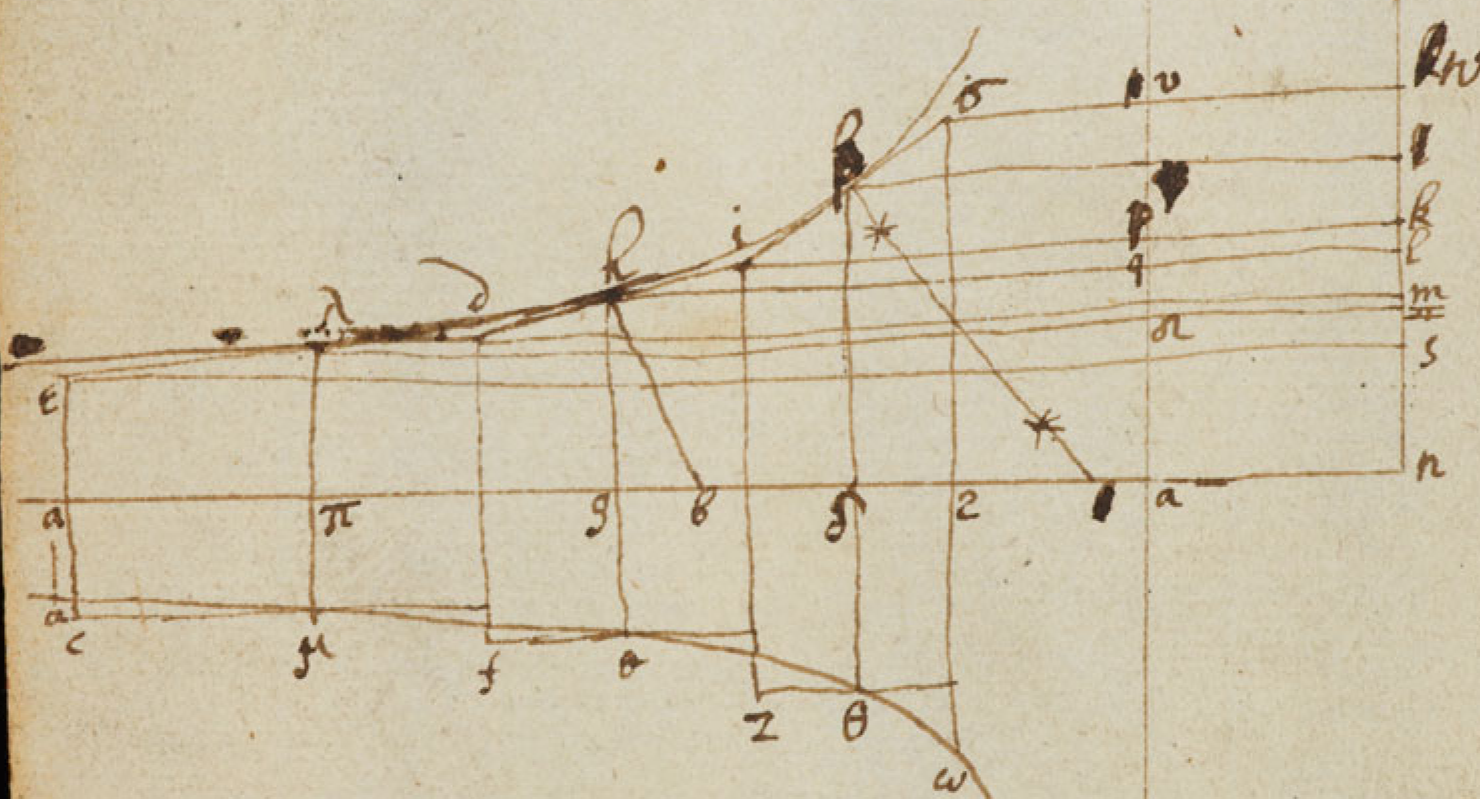
$256\,xy^4 = 81a^5$ ——— $a\sqrt{qq:ax^3}$

$256\,x^5y^4 = a^9$ ——— $\frac{aa}{x}\sqrt{qq:ax^3}$

$256\,x^9y^4 = 625\,a^{13}$ ——— $\frac{a^3}{xx}\sqrt{qq:ax^3}$

$256\,x^{13}y^4 = 6561\,a^{17}$ $\frac{a^4}{x^3}\sqrt{qq:ax^3}$





A Method whereby to square such crooked lines as may be squared.

If y^e crooked lines oka & aεθ are of such a nature that (supposing [gk] parallell to [qa], & [bk] perpendic: to oka at [an] a given line) gk : bg :: an : ge. Then y^e area [age] = [qlna] y^e rectangle made by [an] & [gk].

Demonstration.

Suppose oi, id, de, &c; are tangents of oka, from whose intersections or ends are drawne εc, df, iz, oω, & from whose touch points are drawne βθ, kσ, λμ, &c: all parallel to av. From y^e said intersections draw sw, ik, dm, es, &c. parallel to bn. Since gk : bg :: pd : ip :: an : ge. pd × ge = ir × an. that is □ pkmt = □ uτfs. by y^e same reason tmso = = τrey; & upkw = εuzx &c: Thus also it may be prooved y^t y^e ▭ vwna is equall to any number of such like □s ~~wch~~ inscribed twixt y^e line εω & y^e point a, wch if they be infinite are equall to superficies εaω = vwna. also ~~[illegible]~~ gπμθ. ~~[illegible]~~ = ql = δl. &c.

Prop 1

To find y^e line whose area is exprest by any given equation. Suppose y^e equatio is $\frac{x^3}{a}$. naming y^e quantitys a = an. x = ag. $\frac{x^3}{a}$ = qlna = goa. gk = qa = $\frac{x^3}{aa}$. bk = s. ~~[illegible]~~ ba = v.

$ss - vv + 2vx - xx = \frac{x^6}{a^4}$ wch equatio hath 2 equall rootes & is therefore multiplied according to Huddenius his meth.

$\;\;0\;\;\;0\;\;\;0\;\;\;\;\;\;1\;\;\;\;\;\;2\;\;\;\;\;\;\;\;6$

$vx = x^2 + 3\frac{x^6}{a^4}$. $gb = v - x = \frac{3x^5}{a^4}$. Wherefore if

$\frac{x^3}{aa} : \frac{3x^5}{a^4} :: a : \frac{3xx}{a} = ge$. therefore aεω is a Parab: & age = $\frac{x^3}{a}$ = qlna

Also if y^e Equation be $\frac{a^3}{x}$. Then making $a = an$. $x = ag$. $\frac{a^3}{x} = q\,lna =$ (glea) $qa = \frac{aa}{x} = gh$. $bh = s$. $ba = v$.

$$ss - vv + 2vx - xx = \frac{a^4}{x}$$ w^ch multiplied by Huddenius his method by reason of 2 equall roots.

$$\begin{matrix} 0 & 0 & -1 & -2 & +1 \end{matrix}$$

$$-2vx + 2xx = \frac{2a^4}{x}.$$

$$v = x - \frac{a^4}{x^3}.$$

$x - v = gb = \frac{a^4}{x^3}$. Lastly,

$\frac{aa}{x} : \frac{a^4}{x^3} :: a : \frac{a^3}{xx}$. $\frac{a^3}{xx} = g\varepsilon = y$. & $a^3 = xxy$. w^ch last equation expresseth y^e nature of y^e line $a\varepsilon o$, whose surface $a\varepsilon g = q\,lan = \frac{a^3}{x}$.

Note y^t ~~line~~ I call y^t line [x] to w^ch both y^e lines (or a base) have respect ~~& increaseth & dimi[n]isheth without moving over any superficies~~ as $\pi\alpha$, ga, &c. but y^t line to wch but one line hath respect I call [y] as $g\theta$, $\pi\mu$: or [z] as gh, $\pi\lambda$, &c.

If $ax^m = by^n$. (m & n being numbers y^t signifie y^e dimensions of x & y), then $\frac{nxy}{n+m} = ag\theta$, y^e area of y^e line $a\mu\theta$. And if $a = ~~\ldots~~^n = bx\,x^m \times y^n$. y^n is $\frac{nxy}{n-m} = ag\theta$. y^e area of y^t line.

The squaring of y^e simplest lines in w^ch y is but of one dimension.

Equations expressing y^e nature of y^e lines.	Theire squares.	Lines.	□.
$3xx = ay$. Parab:	$\frac{x^3}{a}$.	$xxy = a^3$	$\frac{a^3}{x}$.
$4x^3 = aay$	$\frac{x^4}{aa}$.	$x^3y = 2a^4$	$\frac{a^4}{xx}$.
$5x^4 = ya^3$	$\frac{x^5}{a^3}$.	$x^4y = 3a^5$	$\frac{a^5}{x^3}$.
$6x^5 = ya^4$	$\frac{x^6}{a^4}$. &c.	$x^5y = 4a^6$	$\frac{a^6}{x7}$. &c.

The square of y^e simplest lines in w^ch y is of 2 dimensions.

The lines squared.	Theire Squares.	Lines squared.	□s.
$4yy = 9ax$. Parab:	$x\sqrt{ax}$.	$4xyy = a^3$	$a\sqrt{ax}$.
$4ayy = 25x^3$	$\frac{xx}{a}\sqrt{ax}$.	$4x^3yy = a^5$	$\frac{aa}{x}\sqrt{ax}$.
$4a^3yy = 49x^5$	$\frac{x^3}{aa}\sqrt{ax}$.	$4x^5yy = 9a^7$	$\frac{a^3}{xx}\sqrt{ax}$.
$4a^5yy = 81x^7$	$\frac{x^4}{a^3}\sqrt{ax}$. &c.	$4x^7yy = 25a^9$.	$\frac{a^4}{x}\sqrt{ax}$ &c.

The square of those lines where y is of 3 dimensions onely.

The lines squared.	Their squares	Lines squared.	Theire Squares
$27xy^3 = 8a^4$	$a\sqrt{c:axx}$.	$27y^3 = 64aax$	$x\sqrt{c:aax}$.
$27x^4y^3 = a^7$	$\frac{aa}{x}\sqrt{c:axx}$.	$27ay^3 = 343x^4$	$\frac{xx}{a}\sqrt{c:aax}$.
$27x^7y^3 = 64a^{10}$	$\frac{a^3}{xx}\sqrt{c:axx}$.	$27a^4y^3 = 1000x^7$	$\frac{x^3}{aa}\sqrt{c:aax}$.
$27x^{10}y^3 = 343a^{13}$	$\frac{a^4}{x^3}\sqrt{c:axx}$. &c	$27a^7y^3 = 2197x^{10}$.	$\frac{x^4}{a^3}\sqrt{c:aax}$ &c
$27xxy^3 = a^5$	$a\sqrt{c:aax}$.	$27y^3 = 125ax^2$	$x\sqrt{c:axx}$.
$27x^5y^3 = 8a^8$	$\frac{aa}{x}\sqrt{c:aax}$.	$27aay^3 = 512x^5$	$\frac{xx}{a}\sqrt{c:axx}$.
$27x^8y^3 = 125a^{11}$	$\frac{a^3}{xx}\sqrt{c:aax}$.	$27a^5y^3 = 1331x^8$	$\frac{x^3}{aa}\sqrt{c:axx}$
$27x^{11}y^3 = 512a^{14}$	$\frac{a^4}{x^3}\sqrt{c:aax}$. &c	$27a^8y^3 = 2744x^{11}$	$\frac{x^4}{a^3}\sqrt{c:axx}$ &c

⊕ And as a man suddainely comeing from greater to lesse light, cannot discerne objects thereby so well, as ~~[illegible]~~ if he came to it by degrees or as when hee hath staid some while in ye lesser light (by reason yt ye motion of ye spirits in ye optick nerve caused by ye greater light doth, untill it bee allayed, ~~[illegible]~~ disturbe & as it were drowne ye ~~motion~~ motion of ye weaker light) soe if the slower motion of ye lower sound immediately succeede ye much more smart motion of ye higher its impression on ye auditory spirits — being then ~~so~~ lesse perceptible, ye lower sound ~~consonance~~ must bee ~~[illegible]~~ less pleasant yn if ~~moved~~ ye step had beene graduated, ~~for that warmth is pleasant to a cold man which is imperceptible to ye same man hot~~ coldness ~~cold~~. Thus a little heate is ~~at ye first im~~ least perceptible to one newly come from a greater. Coroll: 1. The distance of sounds adds to ye imperfection of their concordance. Cor. 2: Tis better to descend yn ascend by leapes ye first makeing ye highest sound harsher, ye seacond makeing ye lower onely lesse perceptible.

⊞ here annex a discourse of ye motion of strings sounding an 8t 5t & 4th & of ye Logarithmes of those strings, or distancess of ye notes.

* these are all ye concords conteined in an Eight. Hereto annex a discourse of ye 3ds & 6ts

The notes in order of concordance

Eight. 5t. 3d maj. 4th. 6t maj. 3d min. 6t min. 2d maj. 7th maj 7th min. 2d min. 5t min.

Of musick.

1. ffirst some one sound must bee pitched upon, to wch all ye musick must bee more especially refered yn to any other sound, (as number to an unit) & let this sound be called ye Cliffe or Key of ye song.

2. Then consider ye sound wch is one or two or thre 8ths above or below ye key (for musick seldome takes a larger compasse yn 3 8ths) The cheife of wch is ye 8th next above ye Key. 3. Each of these Eights are alike divided into pts, for ye pts of ye higher eight are an Eight above their correspondent pts of ye lower eight. so yt ye pts of one Eight knowne give all ye rest, ye other Eights being but a repetition of yt. ~~in a~~ more base or treble sound. (Hence some call an 8th ye largest consonant.) ~~Let this division of an 8th bee called ye Mode of ye song.~~

4. This Eight is first divided into a 5t & 4th, ye fift being next above ye Key; to wch it adds so much sweetnesse yt should this fift bee omitted in any song, ye Key would imparte its name & nature to some sound wch hath a fift above it. And since all harmony wthout a fift is flat, therefore the key must necessarily have a fift above it. ⊞

5. An 8th is next divided into a third major & 6t minor, & lastly into a 3d minor & 6t major. * But as too suddaine a change from lesse to greater light offends ye eye by reason yt ye spirits rarified by ye augmented motion of ye light too violently stretch ye optick nerve: soe ye suddaine passing from grave to acute sounds is not so pleasant as if it were done by degrees, ~~for th~~ because of two greate a change of motion made thereby in ye auditory spirits ⊕. Which graduation may be thus don.

6. The prime pts of an 8th are a 5t & 4th: of a ~~4~~ fift are a 3d major & 3d minor: wch two consist ye first of a tone major & tone minor, ye 2d of a tone major ~~&~~ semitone. A 4th consists of a tone major, minor & semitone. Soe yt an eight consists of thre tone

tone majors, 2 tone minors, & 2 semitones. [The tones might bee againe divided into $\frac{1}{2}$ tones & $\frac{1}{4}$ tones, but they would bee of noe use for tones $\frac{1}{2}$ tones & $\frac{1}{4}$ tones being discords can onely serve to move by from concord to concord wch if done by $\frac{1}{2}$ tones & $\frac{1}{4}$ tones ye number of discords twixt each concords would much more bee harsh yn ye concord would bee pleasant, besides $\frac{1}{2}$ tones & $\frac{1}{4}$ tones are harsher discords by far yn tones, & experience speakes yt an 8th run over by $\frac{1}{2}$ notes is unpleasant. Yet perhaps $\frac{1}{2}$ or $\frac{1}{4}$ notes passed over very hastily wth a larger stay upon ye concords twixt wch they are, might bee delightfull. But since they are such discords, inserted as 'twere by accident, onely to graduate concords, & soe quickly slipt over, ye sence cannot perceive any error or exactnesse in ym, & therefore bee they usefull yet to treate of ym would bee lost labor]

7. The degrees (viz 2 tone majors, a tone minor & semitone in ~~ye~~ 5t & a tone major, a tone minor & semitone in a 4th) are 12 severall ways ordered in ye 8th wch orders are called Modes, generally, because they ~~set~~ much limit ye parts of ye tune from ~~none at to prevent~~ discords sounds of one wth another ~~at ye key, to ye 5t~~ ~~as wth one another~~, particularly because tunes framed by divers of them differ in their aires or Modes.

8. These modes are 3 fold, viz: 6 in wch ye $\frac{1}{2}$ notes are distant 2 tones: foure in wch they are distant one tone: & 2 in wch they are together. The last two are of small or noe use, because every ~~note~~ sound is distant 3 tones from some other, excepting yt there are but 2 fifts. Also thos $\frac{1}{2}$ notes are two harsh to come together much more to bee annext to ye Key or its fift. Neither is the seacond sort very useful for one of ye $\frac{1}{2}$ notes are annexed either to the Key or its 5t or 8t, also 4 of its ~~pts~~ sounds are distant 3 notes from some other: & but 4 of them are distant a fift from some other: whereas there are but 2 in those of ye first sort; distant three notes & sixe of them distant fifts from other sounds; the harshnes of ye $\frac{1}{2}$ notes being there also more moderated by their distance. And therefore ye first 6 are yet in use.

9. The following table may expresse ye 12 Modes in their order of Elegancy. In wch ye tone major & minor are not distinguished, their difference being too little to make new modes by their order changed, though thereby they may add much grace or harshnesse to any particular mode.

	ye Key		2d	3d minor	3d major	4th	Tritonus	5t	6t minor	6t major	7th		8th	
4	O	.	p	.	q	r	.	s	.	t	v	.	o	1
3	s	.	t	v	.	o	.	p	.	q	r	.	s	2
5	r	.	s	.	t	v	.	o	.	p	.	q	r	3
2	p	.	q	r	.	s	.	t	v	.	o	.	p	4
1	t	v	.	o	.	p	.	q	r	.	s	.	t	5
6	v	.	o	.	p	.	q	r	.	s	.	t	v	6
	o	.	p	.	q	r	.	s	x	.	v	.	o	7
	r	.	s	x	.	v	.	o	.	p	.	q	r	8
	s	x	.	v	.	o	.	p	.	q	r	.	s	9
	v	.	o	.	p	.	q	r	.	s	x	.	v	10
	o	.	p	.	q	.	y	s	x	.	v	.	o	11
	s	x	.	v	.	o	.	p	.	q	.	y	s	12

This order may be thus evinced. The first Mode excells ye 2d, by reason of ye ½ note's ~~being~~ more ~~[illegible]~~ convenient placed twixt ye key & its fift, ~~it~~ lesse detracting from ye fift ~~[illegible]~~ because of its greater distance from it. Also ye key hath its 3d major & ye fift its 3d minor in ye 1st mode, but contrarily in ye 2d mode ye key hath its 3d minor & ye 5t its 3d major. The ~~[illegible]~~ sweetness of ye key in ye 3d mode is still more diminished by haveing ye ½ note imediately below it & its 8ts. The 4th Mode succedes as partakeing of ye 3ds deffect; ye sweetnesse of its key's 5t, & consequently of its key, being also diminished by ye ½ note immediately above it. The 5t mode succeds because to ye imperfections of ye 4th this is added yt its first ½ note is next above ye key ~~& its fifts thaw~~ follows. The 6t mode is yet more unpleasant

for both ye key, its 5ts, & eights have a ½ note next below them: Also ye key & its eights have tritones above & below them. Other reasons might bee added for this order, & also for ye order of ye sixt last modes; & it might perhaps bee shown yt ye 7th mode may bee as usefull as ye sixt, but that would bee tedious. Note, yt sometime a note is put out of its place for some particular reason (as to prevent a greater discord &c) but yt seemes soe rare & accidentall to ye songe as not to change its aire or constitute a new mode.

10. The tones major & minor may bee six severall ways in all ye six first modes & but 10 severall ways ordered in each mode. ~~but ye best way generally is to make ye ~~second~~ first third & fift a tone major, the 2d & 4th a tone minor; especially in a solitary voyce.~~ the first is by makeing ye distances, p q, r s, v o, to bee ~~[illegible]~~ tone major, op, & st, to bee tone minors. In this order there are five 5ths, ~~[illegible]~~ 3 third majors, & 3 third minors in an 8th. Thus is ye 3d 5t & first mode best ordered, & thus may ye 4th & 6t moode bee ordered but not ye 2d well, for its keys fift will bee too flat. The 2d way is by putting ye tone minor twixt, o & p, r & s. This order makes also 5 fifts, three 3d majors & 3 3d minors in each 8th. And ~~[illegible]~~ thus may ye 4th, 6t, & 2d Moode bee best ordered; the 3d & 5t moode may bee also ordered thus, but ~~[illegible]~~ ye first not well, for ye keys 5t will bee too flatt. The 3d way is by putting ye minor note betwixt r & s, v & o. & thus each 8th will have five fifts, 2 third majors & 2 minor thirds. The 4th 6t & 2d moode may bee well thus ordered ye 1st & 5t not so well & ye 3d worst of all. The 4th Order is by putting ye minor tone twixt p & q, s & t & thus each 8th hath 5 fifts, 2 minor 3ds, & two major 3ds. Thus may the first, fift, 3d, & 2d moode bee ordered well, but ye 6t & 4th moode not well. The other six orders are lesse convenient to ye Moodes. Note yt, In every 8th there are 6 5ts, 3 major thirds & 4 minor thirds whereof one or more of them are ~~ma~~ too flat or sharpe by about ye 10th pt of a note, but in this computation I onely reckon ye exact concords

Esteeming y^t order ~~most~~ more perfect whose sounds agree in y^e ~~most~~ more & y^e exact concords. Note also y^t every ~~order~~ Eight hath soe many exact 4^ths, 6^t minors & third majors as it hath 5^ts, 3^d majors & 3^d minors their complim^ents to an 8^th.

12 ¶. It may bee required sometimes to raise or let fall y^e voyce in singing w^ch is best done by raising or depressing y^e key of y^e song a fift, (if an 8^t be too greate), for y^t will bee consonant w^th y^e former sound w^ch is now become (for y^e present) gratefull to y^e eare. Also instruments are usually tuned one a fift above another; & a tune (if the keys of severall parts being a fift one above another) might bee pricked for too high a voyce in one pte of y^e Gamut & too base a voyce if removed an 8^th lower. Hence ariseth a comparison of y^e same moode w^th it selfe placed a fift higher. The precedent scheme may serve to represent any of y^e six modes ~~set~~ repeated six times w^th y^e distance of a fift twixt each, according to y^e order of y^e left hand figures. But they cannot bee soe repeated more y^n 3 times, unlesse with more discord y^n harmony.

1. o . p . qr . s . tv . o . p . qr . s . tv . o . p . qr . s .
2. r . s . tv . o . p . qr . s . tv . o . p . qr . s . tv . o .
3. v . o . p . qr . s . tv . o . p . qr . s . tv . o . p . qr .

~~G a b c d e f g a~~

F G, a, b, c, d, e, f, g, a b c d e f g a b c

Any of y^e 6 Moodes (with its eights) may bee represented by any of these 3 orders of letters for y^e key being o they represent y^e first moode, & y^e second it being s, & y^e 3^d if it be r &c: Also y^e first ranke being lowest y^e 2^d a fift above it & y^e 3^d a fift above y^t, this schæme may represent any of y^e Modes w^th y^e same mode one or 2 fifts above or below it.

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11. These degrees have of old beene expressed by ye six notes, vt, re, mi, fa, sol, la, the 7th note being omitted as being a discord to ye key in ye first moode. But of late ye usuall notes are sol, la, mi, fa, sol, la, fa, hitherto expressed by ye letters o, p. q. r. s. t. v. Tis generally best (by see 10) to make ~~from~~ the distance from sol to la, to be a minor tone, from la to mi & fa to sol & mi to fa a major tone, & a semitone from la to fa. Onely in ye 2d Mode make sol & la ~~&~~ mi to bee distant a major tone, fa ~~to be dis~~ a minor tone from sol els ye fift to ye key will bee too flat. Or thus if ye key bee f, a, b, or c make ye distances twixt g & a, c & d to bee a minor tone if ye key bee d or e make ye distances from f to g & c to d a minor tone, but if it be g make ~~a-g~~ a-g=d-c=g-f.

13. Tis usuall to passe from one moode to another in ye midst of a song wch how & to wt moode it may be done will appeare by ye precedent scheme. ffor the 3 rankes may signifie any three ~~A~~ Moodes wch have one common key, as ff is the key of ye first third & sixt Moode, as ye key of ye first 2d & 4th mood &c: And wee may passe from any of those Moodes to another wch in yt scheme have ye same key. But this transition is better done from one key to ye key next it, yn to ye remoter key. Neither may it bee done twixt any other Moodes as twixt ye first & fift or 3d & 4th by reason of their great difference, wch would soe change ye aire of ye song as to make ye pts of it rather seeme divers songs.

14. It may appeare (by sec 10) yt if the key bee f, or b, or e, ye the ~~tra~~ transition may be best done ye degrees of ye moode being ordered ye first way. If ye key bee a or d ye 2d order is best. If ye key bee g ye 3d order is best, & the fourth ye key being c. But in generall, if ye degrees bee ordered ye 4th way in ye 2d Moode & ye 1st way in all ye rest, this transition may ~~will~~ bee well done.

15. From ye consideration of passing from one moode to another in ye same song, two other moodes may bee usefull ye one whereof wants ye key ye other its fift, but these defects are ptly supplyed by the eares retaining the impression of their sweetness made by ye former pte of ye song. g is ye key of one moode & v ye key's 5t in ye other moode.

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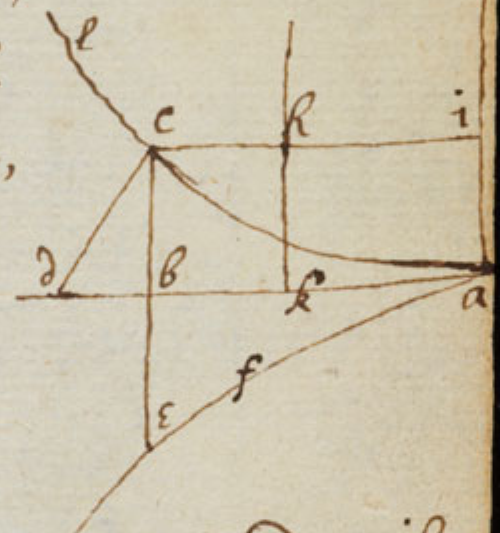
A Method whereby to find y^e areas of those Lines w^ch can bee squared.

Prop: 1st. If $ab=x$. $y=be$. $cb=z$. $bd=v$. secant $=cd$. m & n are numbers expressing y^e dimensions of x, y, or z. a, b, c, d, &c are knowne quantitys, & $\frac{ax^{\frac{m}{n}}}{b}=z$. y^n $\frac{mazx^{\frac{m}{n}}}{nbx}= =\frac{mazx^{\frac{m-n}{n}}}{nb}=v$. And in generall what ever y^e relation twixt x & z bee, make all y^e termes equall to nothing, multiply each terme by so many times zz as x hath dimensions in y^t terme, for a numerator: y^n multiply each terme by soe many times $-x$ as z hath dimensions in y^t terme for a denominator in y^e valor of v.



Prop: 2d. If $ki=r$. & $rv=zy$. y^n ki & be describe equall spaces $kigk$, or $kiak$ & $abef$. ~~that~~ is $abef=aik$

Prop: 3d. If $a^n x^m = b^n y^n$. Or $\frac{ax^{\frac{m}{n}}}{b}=y$. y^n is $\frac{nxy}{n+m}= \frac{nax^{\frac{m+n}{n}}}{nb+mb}=abef$ y^e area of y^e line aef. And if $\frac{a}{bx^{\frac{m}{n}}}=y$: y^n is $\frac{nxy}{n-m}=\frac{na}{\overline{n-m}\times bx^{\frac{-n+m}{n}}}=abef=\frac{na}{\overline{n-m}\times bx^{\frac{m-n}{n}}}$

Demonstration.

ffor ~~$\frac{ax^{\frac{m}{n}}}{b}=(sq)y$. Therefor~~ Suppose $akki$ is a parallelogram & equall to $\frac{nax^{\frac{m+n}{n}}}{nb+mb}$. y^n is $\frac{nax^{\frac{m+n}{n}}}{nbr+mbr}=ai=z$. & (prop 1) $\frac{az^2x^{\frac{m+n}{n}}}{brxz}=\frac{azx^{\frac{m}{n}}}{br}=v$. & (prop 2d) $rv=zy$, y^t is $\frac{ax^{\frac{m}{n}}}{b}=y$.

P

Prop: 4th. If $y = ax^m + bx^n$. $\overset{n}{y}$ is $\frac{ax^{m+1}}{m+1} + \frac{bx^{n+1}}{n+1} = abes$

And in generall if y^e valor of y consists of severall termes so y^t x ~~hath more of one dimensions in y^e denominator of any terme, or~~ is not of divers dimensions in y^e denominator of any terme, y^n multiply each terme by x & divide it by y^e number of y^e dimensions of x, all those products shall bee y^e areas of y^e given line: supposeing also y^t either none or all y^e signes of those termes are changed by this operation. ffor if some bee changed & others bee not they proceed divers ways & joyne not ~~y^e area is infinite~~, & y^n y^e quantitys [blot] y or x must bee increased or diminished or otherwis altered.

The reason of this prop: is, y^t y^e area described by y is also described by its parts y^t is by y^e termes of its valor, & w^t areas those termes describe appeares by prop 3^d.

Prop 5t. The progressions in this Table may bee designed by these geomet: lines. Whereby also any intermediate termes may bee found.

b a b b b b b

1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	$\sim 1 = y$.
-2.	-1.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	$\sim x = y$.
3.	1.	0.	0.	1.	3.	6.	10.	15.	21.	28.	36.	$\sim xx - x = 2y$.
-4.	-1.	0.	0.	0.	1.	4.	10.	20.	35.	56.	84.	$\sim x^3 - 3xx + 2x = 6y$.
5.	1.	0.	0.	0.	0.	1.	5.	15.	35.	70.	126.	$\sim x^4 - 6x^3 + 11xx - 6x = 24y$.
-6.	-1.	0.	0.	0.	0.	0.	1.	6.	21.	56.	126.	$\sim x^5 - 10x^4 + 35x^3 - 50x^2 + 24x = 120y$.
7.	1.	0.	0.	0.	0.	0.	0.	1.	7.	28.	84.	$\sim x^6 - 15x^5 + 85x^4 - 225x^3 + 274xx - 120x = 720y$
-8.	-1.	0.	0.	0.	0.	0.	0.	0.	1.	8.	36.	$\sim x^7 - 21x^6 + 175x^5 - 735x^4$ &c: $= 5040y$.

The distance of y^e terme b from y^e terme a being called x. & the quantity of y^t termes being y. & each terme being distant an unit from y^e next. The nature of w^ch table is such y^t y^e summe of any figure & y^e figure above it is equall to y^e figure after it. & y^e nature of y^e lines are such y^t ~~y^e precedent~~ any figure, multiplyed by y^e ~~dimen~~ number of dimensions of x in y^e first terme, being substracted from y^e figure following it, is equall to y^e figure under y^t following figure. And y^t y^e numbers of y may be deduced hence $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7$ &c.

Prop 6t. If $\frac{m}{n}=x$. This Progression $\frac{n\times m\times\overline{m-n}\times\overline{m-2n}\times\overline{m-3n}\times\overline{m-4n}\times\overline{m-5n}}{n\times n\times 2n\times 3n\times 4n\times 5n\times 6n}$ &c gives all ye quantitys downward, in ye preceding table. As if $m=3$. $n=1$. the quantitys downward are $\frac{1}{1}.\ \frac{m}{n}.\ \frac{m\times\overline{m-n}}{n\times 2n}.\ \frac{m\times\overline{m-n}\times\overline{m-2n}}{n\times 2n\times 3n}.\ \frac{m\times\overline{m-n}\times\overline{m-2n}\times\overline{m-3n}}{n\times 2n\times 3n\times 4n}$. &c yt is 1.3.3.1.0.&c So if $\frac{m}{n}=\frac{1}{2}=x$. yt $1.\ \frac{1}{2}.\ -\frac{1}{8}.\ \frac{1}{16}.\ -\frac{5}{128}.\ \frac{7}{256}$. &c. are ye terms downward.

Prop 7th. $\overline{a+b}|^{\frac{m}{n}}=a^{\frac{m}{n}}+\frac{m}{n}\times\frac{b}{a}\times a^{\frac{m}{n}}+\frac{m}{n}\times\frac{m-n}{2n}\times\frac{bb}{aa}\times a^{\frac{m}{n}}+\frac{m}{n}\times\frac{m-n}{2n}\times\frac{m-2n}{3n}\times\frac{b3}{a3}\times a^{\frac{m}{n}}$. &c As may bee deduced from $a^{\frac{m}{n}}\times\frac{mb}{na}\times\frac{m-n\times b}{2na}\times\frac{m-2n\times b}{3na}\times\frac{m-3n\times b}{4na}\times\frac{mb-4nb}{5na}$ &c.

~~Prop 8th $\frac{c}{\overline{a+b}|^{\frac{m}{n}}}=\frac{c}{a^{\frac{m}{n}}}$~~ The truth of this Prop: appeareth by compareing it wth ye two former as also by calculation if $\frac{m}{n}$ is a whole & affirmative number, or b ~~never~~ lesse yn ~~a~~ ~~unit~~.

Prop 8th. $\frac{1}{\overline{a+b}|^{\frac{m}{n}}}=\frac{1}{a^{\frac{m}{n}}}-\frac{m}{n}\times\frac{b}{a}\times\frac{1}{a^{\frac{m}{n}}}.-\frac{m}{n}\times\frac{-m-n}{2n}\times\frac{bb}{aa}\times\frac{1}{a^{\frac{m}{n}}}-\frac{m}{n}\times\frac{-m-n}{2n}\times\frac{-m-2n}{3n}\times\frac{b3}{a3}\times\frac{1}{a^{\frac{m}{n}}}$. &c. As may bee deduced from $\frac{1}{a^{\frac{m}{n}}}\times\frac{-mb}{na}\times\frac{-mb-nb}{2na}\times\frac{-mb-2nb}{3na}\times\frac{-mb-3nb}{4na}$ &c.

The truth of this appeares also by ye 5t & 6t propositions, or by calculation If $a \sqsubset b$.

The truth of these two prop: is also thus demonstrated If $\overline{a+b}|^{\frac{-1}{1}}=\frac{1}{a+b}$ & divide, 1, by $a+b$ as in decimall fractions & find ye quote $\frac{1}{a}-\frac{b}{aa}+\frac{bb}{a3}-\frac{b3}{a4}+\frac{b4}{a5}$ &c as appeareth also by multiplying both pts by $a+b$. So If extract ye root of a^2+b as if they were decimall numbers & find $\sqrt{a^2+b}=a+\frac{b}{2a}-\frac{bb}{8a3}+\frac{b3}{16a5}$ &c, as also may appeare by squareing both pts.

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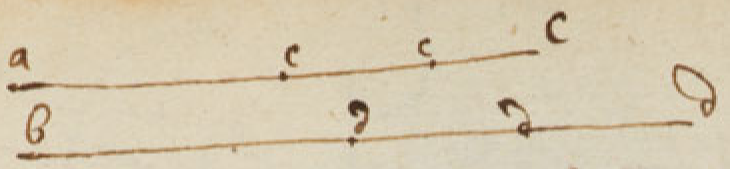


1. If two bodys c, d describe ye streight lines ac, bd, in ye same time, (calling $ac = x$, $bd = y$, $p =$ motion of c, $q =$ motion of d) & if I have an equation expressing ye relation of $ac = x$ & $bd = y$ whose termes are all put equall to nothing. I multiply ~~ye equation~~ each terme of yt equation by so many times py or $\frac{p}{x}$ as x hath dimensions in it, & ~~again~~ also by soe many times qx or $\frac{q}{y}$ as y hath dimensions in it. the sume of these products is an equation expressing ye ~~motions~~ relations of ye motions of c & d. Example if $ax^3 + a^2yx - y^3x + y^4 = 0$. yn ~~[illegible]~~ $3apxx + a^2py - py^3 + aaqx - 3qyyx + 4qy^3 = 0$.

2. If an equation expressing ye relation of their motions bee given, tis more difficult & sometimes Geometrically impossible, thereby to find ye relation of ye spaces described by those motions.

If $apx^{\frac{m}{n}}=q$. —— then $\frac{na}{m+n}x^{\frac{m+n}{n}}=y$.

As if $m=3$. $n=2$. y^n $apx^{\frac{3}{2}}=q$, & $\frac{2a}{3}x^{\frac{5}{2}}=y$. Or if $apx^{\frac{3}{2}}=q$ $=\frac{ap}{x^{\frac{3}{2}}}$, y^n $m=-3$. $n=2$. & $\frac{2a}{-1}x^{\frac{-1}{2}}=\frac{-2a}{x^{\frac{1}{2}}}=y$. If y^e valor of q consisteth of severall such termes, consider each terme severally. as if $ax+bxx=q$. y^e first terme gives $\frac{ax^2}{2}$ y^e 2d $\frac{bx^3}{3}$. therefore $\frac{axx}{2}+\frac{bxxx}{3}=y$.

In generall multiply y^e valor of q by x & divide ~~by y^e logar~~ each terme of it by y^e logarithme of x, in y^t terme: if y^t ~~equation~~ valor of q consist of simple termes.

$$\frac{\overline{max^m+nbx^n}\times\overline{\delta x^r+\varepsilon x^s}-\overline{r\delta x^r-s\varepsilon x^s}\times\overline{ax^m+bx^n}}{x\times\overline{\delta x^r+\varepsilon x^s}\times\overline{\delta x^r+\varepsilon x^s}}=\frac{q}{p}.\quad\frac{ax^m+bx^n}{\delta x^r+\varepsilon x^s}=y.$$

$$\frac{-r\delta x^{r-1}}{\delta\delta x^{2r}+2\delta\varepsilon x^r+\varepsilon\varepsilon}=\frac{q}{p}.\qquad\frac{2}{\delta x^r+\varepsilon}=y.$$

$$\frac{\overline{m-r}\times a\delta x^{m+r}+\overline{m-s}\times a\varepsilon x^{m+s}+\overline{n-r}\times b\delta x^{n+r}+\overline{n-s}\times b\varepsilon x^{n+s}}{x \text{ in } \delta\delta x^{2r}+2\delta\varepsilon x^{r+s}+\varepsilon\varepsilon x^{2s}}=\frac{q}{p}.\quad\frac{ax^m+bx^n}{\delta x^r+\varepsilon x^s}=y.$$

~~$mb+\overline{3n-2m}\times cx$~~

$$\frac{ma+\overline{3n-2m}\times bx^{n-m}}{x}\times\sqrt{ax^m+bx^n}=\frac{q}{p}.\quad\overline{a+bx^{n-m}}\times\sqrt{ax^m+bx^n}=y.$$

or thus

$$\frac{ma+\overline{3n+m}\times bx^n}{2x}\times\sqrt{ax^m+bx^{n+m}}=\frac{q}{p}.\quad\overline{a+bx^n}\sqrt{ax^m+bx^{n+m}}=y.$$

$$\frac{\overline{mm+8mn+15nn}\times\delta\delta x^{2n}\ \overline{{}^{2}mn-mm}\times b\varepsilon\varepsilon}{x}\sqrt{\varepsilon x^m+\delta x^{m+n}}=\frac{q}{p}.$$ And y^n

$$\overline{2m+6n}\times\delta\delta x^{2n}+2n\delta\varepsilon x^n-\overline{2m-4n}\times\varepsilon\varepsilon\times\sqrt{\varepsilon x^m+\delta x^{m+n}}=y.$$

Or thus

$$\frac{\overline{3m-2n}\times maax^{m-n}+\overline{3n-2m}\times nbbx^{n-m}}{2x}\sqrt{ax^m+bx^n}=\frac{q}{p}.$$

$$\overline{maax^{m-n}+\overline{m-n}\times ab-nbx^{n-m}}\sqrt{ax^m+bx^n}=y.$$

$+$

$\frac{mb-nb}{m}+ax^{n-m}-\frac{rbb}{ma}x^{r-n}\times\sqrt{cx^m+dx^n}=y$. And

$\frac{3r-2m\times adx^{r-m}+3m+2r\times\frac{rbb}{ma}x^{m-r}}{2x}\times\sqrt{cx^m+dx^n}=\frac{q}{p}$.

$mb-nab+maax^{n-m}+nbbx^{m-n}\sqrt{x^m+dx^n}=y$. And

$3n-2m\times naax^{n-m}+2n-3m\times mbb\times\sqrt{aadx^n-bbx^m}=\frac{q}{p}$.

$=y$.

$maax^{n-m}+m-n\times ab-mbbx^{m-n}\sqrt{aax^n-abx^n}$

$maax^{m-n}+m-n\times ab-nbbx^{n-m}\sqrt{ax^m+bx^n}=y$. And

$\frac{3m-2n\times maax^{m-n}+3n-2m\times nbbx^{n-m}}{2x}\sqrt{ax^m+bx^n}=\frac{q}{p}$.

$\frac{mac+3r-2m\times adx^{r-m}+3m+2n\times bcx^{m+n}+3r+2n\times bdx^{n+r}}{2x}$ in $\sqrt{cx^m+dx^r}=\frac{q}{p}$.

$ac+adx^{r-m}+bcx^{m+n}+bdx^{r+n}\times\sqrt{cx^m+dx^r}=y$

$\frac{3m-2n\times 2n-m\times md^3x^{m-n}+3n-2m\times 5n-4m\times n\varepsilon^3x^{2n-2m}}{2x}$ in $\sqrt{dx^m+\varepsilon x^n}=\frac{q}{p}$

$\frac{2n-m\times md^3x^{m-n}+2n-m\times m-n\times\varepsilon dd+n-m\times n\varepsilon\varepsilon dx^{n-m}+3n-2m\times n\varepsilon^3x^{2n-2m}}{}\times$

$\times\sqrt{dx^m+\varepsilon x^n}=y$.

Or more ~~generally~~ generally,

$\frac{3m-2n\times mcddx^{m-n}+2m-3n\times nc\varepsilon\varepsilon x^{n-m}+3m+2p\times mbddx^{m+p}+3n+2p\times b\varepsilon dmx^{n+p}}{2x}$ in

in $\sqrt{dx^m+\varepsilon x^n}=\frac{q}{p}$. And $mddcx^{m-n}+m-n\times cd\varepsilon$

$-nc\varepsilon\varepsilon x^{n-m}+mbddx^{p+m}+mbd\varepsilon x^{n+p}\sqrt{dx^m+\varepsilon x^n}=y$.

$\frac{9m-2n}{m}\times 2m+n\times\varepsilon^3x^{2m-n}\quad\frac{+6n-3m}{4n-m}\times 9nd^3x^{2n-m}$ in $\sqrt{\frac{2m+n}{m}\times\varepsilon\varepsilon x^m}+$

$+\frac{9ndd}{4n-m}x^{2n-m}-3d\varepsilon x^n=\frac{2qx}{p}$. And $\frac{2m+n}{m}\times\varepsilon^3x^{2m-n}+\frac{n-m}{m}\times\varepsilon\varepsilon dx^m+$

$+\frac{m-n}{4n-m}\times 3dd\varepsilon x^n\quad+\frac{9nd^3}{4n-m}\times x^{2n-m}$ in $\sqrt{\frac{2m+n}{m}\times\varepsilon\varepsilon x^m-3d\varepsilon x^n+\frac{9ndd}{4n-m}x^{2n-m}}=$

$=y$.

$$\frac{\overline{5n-2m}\times\overline{2m-5n}\times 3mnn\,b^5}{\overline{2n+m}\times\overline{16n^4-8nnmm+m^4}}\times x^{3m-2n}+\frac{\overline{2m-5n}\times 3mn\,b^4c}{\overline{2n+m}\times\overline{4nn-mm}}x^{2m-n}$$

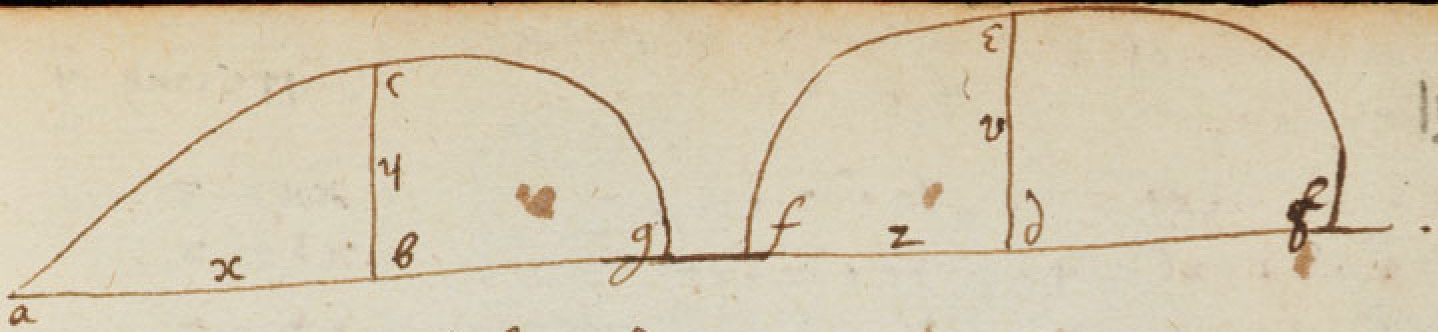
$$+\frac{\overline{2n-2m}\times\overline{5n-2m}\times nb^3cc}{\overline{2n+m}\times\overline{4nn-mm}}x^{m}+\frac{\overline{2n-2m}}{2n+m}\times bbc^3x^{n}+\frac{4m-n}{5n-2m}\times bc^4x^{2n-m}$$

$$+\frac{3m-6n}{5n-2m}\times c^5x^{3n-2m}\ \text{in}\ \sqrt{\frac{\overline{5n-2m}\times nbb}{4nn-mm}x^{m}+bcx^{n}+ccx^{2n-m}}=\cancel{f}$$

$=y.$ And, $\frac{\overline{7m-4n}\times\overline{5n-2m}\times\overline{2m-5n}\times 3mnn\,b^5}{\overline{2n+m}\times\overline{16n^4-8mmnn+m^4}}\times x^{3m-2n}$

$$+\frac{\overline{4m-n}\times\overline{2m-5n}\times 3mn\,b^4c}{\overline{2n+m}\times\overline{4nn-mm}}x^{2m-n}+\frac{\overline{8n-5m}\times\overline{3m-6n}}{5n-2m}\times c^5x^{3n-2m}$$

$$\text{in}\ \sqrt{\frac{\overline{5n-2m}\times nbb}{4nn-mm}x^{m}+bcx^{n}+ccx^{2n-m}}=\frac{2qx}{p}.$$



sit $ab=x$. $bc=y$. $df=z$, $de=v$.

The area abc of ye line whose nature is	is equall to ye area fde of ye line whose nature is	supposeing ye relation twixt ab & fd to bee
$2xx\sqrt{c+dxx}=y$.	$\sqrt{cz+dzz}=v$.	$xx=z$.
$\sqrt{cx+dxx}=y$.	$a\sqrt{caz+daazz}=v$.	$ax=z$.
$-\frac{1}{x^3}\sqrt{cx+d}=y$.	$\sqrt{cz+dzz}=v$.	$1=zx$.
$-\frac{2}{x^5}\sqrt{cxx+d}=y$.	$\sqrt{cz+dzz}=v$.	$1=zx^2$.
$-\frac{3}{x^7}\sqrt{cx^3+d}=y$.	$\sqrt{cz+dzz}=v$.	$1=zx^3$.
$3x^3\sqrt{cx+dx^4}=y$.	$\sqrt{cz+dzz}=v$.	$x^3=z$.
$4x^5\sqrt{c+dx^4}=y$.	$\sqrt{cz+dzz}=v$.	$x^4=z$.
$\frac{1}{2x}\sqrt{cx^{\frac{3}{2}}+dxx}=y$.	$\sqrt{cz+dzz}=v$.	$x=zz$
In generall $nx^{n-1}\times\sqrt{cx^n+dx^{n+n}}=y$.	$\sqrt{cz+dzz}=v$.	$x^n=z$.
$2x\sqrt{c+dx^4}=y$.	$\sqrt{c+dzz}=v$.	$xx=z$
$3xx\sqrt{c+dx^6}=y$	$\sqrt{c+dzz}=v$	$x^3=z$.
$-\frac{1}{x^3}\sqrt{cxx+d}=y$.	$\sqrt{c+dzz}=v$	$1=zx$.
$-\frac{2}{x^5}\sqrt{x^4+d}=y$.	$\sqrt{c+dzz}=v$	$1=zxx$
$\frac{1}{2x}\sqrt{cx+dxx}=y$.	$\sqrt{c+dzz}=v$.	$x=zz$.
$\frac{3}{2}\sqrt{cx+dx^4}=y$.	$\sqrt{c+dzz}=v$.	$x^3=zz$
$-\frac{1}{2xx}\sqrt{cx+d}=y$.	$\sqrt{c+dzz}=v$.	$1=xzz$.
$-\frac{3}{2x^4}\sqrt{cx^3+d}=y$.	$\sqrt{c+dzz}=v$.	$1=x^3zz$.
In generall $nx^{n-1}\sqrt{c+dx^{n+n}}=y$.	$\sqrt{c+dzz}=v$.	$x^n=z$.

The area abc of ye line ~ .	is equall to ye area of ye line ~	Supposing ye
$2x\sqrt{c+\partial xx+\varepsilon x^4}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$xx=z.$
$3xx\sqrt{c+\partial x^3+\varepsilon x^6}=y.$	$\sqrt{c+\partial z+\varepsilon z^2}=v.$	$x^3=z.$
$4x^3\sqrt{c+\partial x^4+\varepsilon x^8}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$x^4=z.$
$\frac{-1}{x^3}\sqrt{cxx+\partial x+\varepsilon}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$1=zx.$
$\frac{-2}{x^5}\sqrt{cx^4+\partial xx+\varepsilon}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$1=zxx.$
$-\frac{3}{x^7}\sqrt{cx^6+\partial x^3+\varepsilon}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$1=zx^3.$
$\frac{1}{2x}\sqrt{cx+\partial x^{\frac{3}{2}}+\varepsilon xx}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$x=zz.$
$\frac{-1}{2xx}\sqrt{cx+\partial x^{\frac{1}{2}}+\varepsilon}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$1=zzx.$
In generall.		
$nx^{n-1}\sqrt{c+\partial x^n+\varepsilon x^{n+n}}=y.$	$\sqrt{c+\partial z+\varepsilon zz}=v.$	$x^n=z.$
$\frac{b}{a+bx}=y.$	$\frac{1}{z}=v$	$a+bx=z.$
$\frac{2bx}{a+bxx}=y.$	$\frac{1}{z}=v$	$a+bx^2=z.$
$\frac{3bxx}{a+bx^3}=y.$	$\frac{1}{z}=v$	$a+bx^3=z$
$\frac{4bx^3}{a+bx^4}=y.$	$\frac{1}{z}=v$	$a+bx^4=z.$
$\frac{-b}{axx+bx}=y$	$\frac{1}{z}=v$	$ax+b=zx$
$\frac{-2b}{ax^3+bx}=y$	$\frac{1}{z}=v$	$ax^2+b=zx^2.$
$\frac{-3b}{ax^4+bx}=y.$	$\frac{1}{z}=v$	$ax^3+b=zx^3.$
$\frac{b}{2a\sqrt{x}+2bx}=y$	$\frac{1}{z}=v$	$a+b\sqrt{x}=z.$
$\frac{3xb}{2a\sqrt{x}+2bxx}=y$	$\frac{1}{z}=v$	$a+bx^{\frac{3}{2}}=z.$
$\frac{-b}{ax\sqrt{x}+bx}=y$	$\frac{1}{z}=v$	$a+\frac{b}{\sqrt{x}}=z.$
$\frac{-3b}{axx\sqrt{x}+bx}=y$	$\frac{1}{z}=v$	$a+\frac{b}{x\sqrt{x}}=z.$
$\frac{a+2bx}{ax+bxx}=y.$	$\frac{1}{z}=v.$	$ax+bx^2=z.$
$\frac{a+3bxx}{ax+bx^3}=y.$	$\frac{1}{z}=v.$	$ax+bx^3=z.$

$\frac{axx-b}{ax^3+bxx}=y$. —— $\frac{1}{z}=v$. —— $aax+b=zx$.

$\frac{2a+3bx}{ax+bxx}=y$. —— $\frac{1}{z}=v$. —— $ax^2+bx^3=z$.

$\frac{2a+4bxx}{ax+bx^3}=y$. —— $\frac{1}{z}=v$. —— $axx+bx^4=z$

$\frac{2a+5bx^3}{ax+bx^4}=y$. —— $\frac{1}{z}=v$. —— $ax^2+bx^5=z$

$\frac{3a+4bx}{ax+bx^2}=y$. —— $\frac{1}{z}=v$. —— $ax^3+bx^4=z$

$\frac{3a+5bxx}{ax+bx^3}=y$. —— $\frac{1}{z}=v$. —— $ax^3+bx^5=z$.

$\frac{4a+5bx}{ax+bxx}=y$ —— $\frac{1}{z}=v$ —— $ax^4+bx^5=z$

$\frac{4a+6bxx}{ax+bx^3}=y$ —— $\frac{1}{z}=v$ —— $ax^4+bx^5=z$.

$\frac{-a+bxx}{ax+bx^3}=y$. —— $\frac{1}{z}=v$ —— $a+bx^2=xz$

$\frac{-a+2bx^4}{axx+bx^5}=y$. —— $\frac{1}{z}=v$. —— $a+bx^3=xxz$.

$\frac{-2a-bx}{ax+bxx}=y$. —— $\frac{1}{z}=v$ —— $a+bx=xxz$.

In generall.

$\frac{madx^{m-1}+nbdx^{n-1}}{ax^m+bx^n}=y$. $\frac{d}{z}=v$. $ax^m+bx^n=z$

Note y^t these are compounded onely of y^e first simplest Areas:

The area abe of y^e line	is equall to y^e area fde of y^e line	Supposeing that
$\frac{xx}{\sqrt{dxx-dc}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{xx}{d}-\frac{c}{d}}=z.$
$\frac{x}{2\sqrt{dxx-dcx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{x-c}{d}}=z.$
$\frac{-1}{2xx\sqrt{d-cdx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{1-cx}{dx}}=z$
$\frac{-1}{x^3\sqrt{d-cdxx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{1-cxx}{dxx}}=z.$

Or generally.

$\frac{sx^{3s-1}}{\sqrt{dx^{2s}-cd}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{\frac{x^{2s}-c}{d}}=z.$
$\frac{bcc+2bbcdx+b^3ddxx}{\sqrt{2bcx+bbdxx}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2cbx+dbbxx}=z$
$\frac{2bcc+4bbcdxx+2b^3ddx^4}{\sqrt{2bc+bbdxx}}=y.$	$\sqrt{cc+dzz}=v.$	$x\sqrt{2bc+bbdxx}=z.$
$\frac{-bccxx-2bbcdx-b^3dd}{x^3\sqrt{2bcx+bbd}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2cbx+bbd}=zx.$
$\frac{-2bccx^4-4bbcdxx-2b^3dd}{x^5\sqrt{2bcxx+bbd}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2bcxx+bbd}=zxx.$

In generall

$\frac{mbccx^m+2mbbcdx^{2m}+mb^3ddx^{3m}}{x\sqrt{2bcx^m+dbbx^{2m}}}=y.$	$\sqrt{cc+dzz}=v.$	$\sqrt{2bcx^m+bbdx^{2m}}=z.$
$\frac{b\sqrt{c+ad+bdx}}{2\sqrt{a+bx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+bx}=z.$
$\frac{bx\sqrt{c+ad+bdxx}}{\sqrt{a+bxx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+bxx}=z.$
$\frac{-b\sqrt{cx^2+adxx+bdx}}{2xx\sqrt{axx+bx}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+\frac{b}{x}}=z.$
$\frac{-b\sqrt{cxx+adxx+bd}}{x^3\sqrt{axx+b}}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{a+\frac{b}{xx}}=z.$

In generall.

$\frac{max^m+nbx^n}{2x\sqrt{ax^m+bx^n}}\times\sqrt{c+adx^m+bdx^n}=y.$	$\sqrt{c+dzz}=v.$	$\sqrt{ax^m+bx^n}=z.$

$\frac{\varepsilon b\sqrt{ac+d\varepsilon\varepsilon+cbx}}{2a+2bx\times\sqrt{a+bx}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon}{\sqrt{a+bx}}=z$.

$\frac{\varepsilon bx\sqrt{ac+d\varepsilon\varepsilon+cbxx}}{a+bxx\sqrt{a+bxx}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon}{\sqrt{a+bxx}}=z$.

$\frac{-\varepsilon b\sqrt{acxx+d\varepsilon\varepsilon xx+cbx}}{2ax^2+2bx\sqrt{axx+bx}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon x}{\sqrt{axx+bx}}=z$.

$\frac{-\varepsilon b\sqrt{acxx+d\varepsilon\varepsilon xx+cb}}{ax^3+bx\sqrt{axx+b}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon x}{\sqrt{axx+b}}=z$.

$\frac{4a^4cdxx+4aabcdx+bbcd}{2aaxx+2bx\sqrt{4a^4ddxx+4aabddx}}=y$. $\sqrt{cc+ddzz}=v$. $\frac{cb}{2ad\sqrt{aax^2+bx}}=z$

$\frac{4a^4cdx^4+4aabcdxx+bbcd}{aax^3+bx\times 2ad\sqrt{aax^4+bxx}}=y$. $\sqrt{cc+ddzz}=v$. $\frac{cb}{2adx\sqrt{a^2x^2+b}}=z$

$\frac{-4a^4cd-4aabcdx-bbcdxx}{2aax+2bxx\times 2ad\sqrt{aa+bx}}=y$. $\sqrt{cc+ddzz}=v$. $\frac{cbx}{2ad\sqrt{aa+bx}}=z$.

$\frac{-4a^4cdx^2-4aabcdx^4-bbcdx^6}{aa\text{[illegible]}+bxx\times 2ad\sqrt{aa+bxx}}=y$. $\sqrt{cc+ddzz}=v$. $\frac{cbxx}{2ad\sqrt{aa+bxx}}=z$.

In Generall.

$\frac{\varepsilon max^m+\varepsilon nbx^n\sqrt{cax^m+cbx^n+d\varepsilon\varepsilon}}{2ax^{m+1}+2bx^{n+1}\times\sqrt{ax^m+bx^n}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon}{\sqrt{ax^m+bx^n}}=z$.

$\frac{\varepsilon max^{m-1}+\varepsilon nbx^{n-1}\sqrt{cax^m+cbx^n+d\varepsilon\varepsilon}}{2aax^{2m}+4abx^{m+n}+2bbx^{2n}}=y$. $\sqrt{c+dzz}=v$. $\frac{\varepsilon}{\sqrt{ax^m+bx^n}}=z$.

$\frac{c\varepsilon b^3xx}{aa+2abxx+bbx^4}=y$. $\sqrt{cc-\frac{acc}{\varepsilon\varepsilon}zz}=v$. $\frac{\varepsilon}{\sqrt{a+bbxx}}=z$.

In Generall

$\frac{nb^3c\varepsilon x^{\frac{3n-2}{2}}}{2aa+4abx^n+2bbx^{2n}}=y$. $\sqrt{cc-\frac{acc}{\varepsilon\varepsilon}zz}=v$. $\frac{\varepsilon}{\sqrt{a+bbx^n}}=z$.

$\sqrt{c+\partial}$.

$\frac{[illegible]}{a\partial}$

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$\frac{cb}{a+bx}=y.$ —— $c+zv=0.$ —— $\frac{1}{a+bx}=z$

$\frac{2bcx}{a+bxx}=y.$ $c+zv=0.$ $\frac{1}{a+bxx}=z$

$\frac{3bcxx}{a+bx^3}=y.$ $c+zv=0.$ $\frac{1}{a+bx^3}=z.$

$\frac{cb}{axx+bx}=y.$ —— $c=zv.$ —— $\frac{x}{ax+b}=z.$

$\frac{cb}{2ax^3+^2bx}=y.$ —— $c=zv.$ —— $\frac{xx}{ax^2+b}=z.$

As before, In generall,

$\frac{cmax^{m-1}+ncbx^{n-1}}{ax^m+bx^n}=y.$ $c+zv=0.$ —— $\frac{1}{ax^m+bx^n}=z.$

Also

$\frac{rdx^{r-1}+sex^{s-1}\times\overline{ax^m+bx^n}}{ax^m+bx^n\times\overline{dx^r+ex^s}}\ \frac{-max^{m-1}-nbx^{n-1}}{ax^m+bx^n}=y.$ $c=zv.$ $\frac{dx^r+ex^s}{ax^m+bx^n}=z.$

$\frac{crdx^{r-1}+csex^{s-1}}{dx^r+ex^s}\ \frac{-cmax^{m-1}-cnbx^{n-1}}{ax^m+bx^n}=y.$ $c=zv.$ $\frac{dx^r+ex^s}{ax^m+bx^n}=z.$

$\frac{-9ac}{2bx^4+^2cx}=y.$ $a=zv.$ $\overline{bx^3+c}\times\sqrt{bx^4+cx}=x^5z.$

$\frac{-3ac}{bx^3+cx}=y.$ $a=zv.$ $\overline{bxx+c}\sqrt{bxx+c}=x^3z.$

$\frac{-3ac}{2bxx+2cx}=y.$ $a=zv.$ $\overline{bx+c}\sqrt{bxx+cx}=xxz.$

$\frac{3ac}{2b+^2cx}=y.$ $a=zv.$ $\overline{b+cx}\sqrt{b+cx}=z.$

$\frac{3acx}{b+cxx}=y.$ $a=zv.$ $\overline{b+cxx}\sqrt{b+cxx}=z$

As before was found, In generall

$\frac{\overline{3m+2r}\times abx^{m+r}+\overline{3n+2r}\times acx^{n+r}}{2bx^{m+r+1}+2cx^{n+r+1}}=y.$ $az=v.$

And $\overline{bx^{m+r}+cx^{n+r}}\times\sqrt{bx^m+cx^n}=z.$

$$xx=ay. \qquad 1=v. \qquad x^3=3az$$
$$x^3=ay. \qquad 1=v. \qquad x^4=4az$$
$$x^4=ay. \qquad 1=v. \qquad x^5=5az.$$ 160
$$a=xxy. \qquad 1=v. \qquad -a=xz.$$
$$a=x^3y. \qquad 1=v. \qquad -a=2xxz.$$
$$x^3=ayy. \qquad 1=v. \qquad 4x^5=25azz.$$
$$a=x^3yy. \qquad 1=v. \qquad 4a=zzx.$$

In generall

$$ax^m=y. \qquad 1=v. \qquad \frac{a}{m+1}x^{m+1}=z.$$

That is.

multiply y^e valor of y. by x, & ~~each~~ divide each terme in y^t valor by soe many units as x hath dimensions in y^t terme, y^e product is y^e area.

$$\frac{c}{bb+2bcx+ccxx}=y. \qquad 1=v. \qquad \frac{1}{b+cx}=z.$$
$$\frac{2cx}{bb+2bcxx+ccx^4}=y. \qquad 1=v. \qquad \frac{1}{b+cxx}=z.$$
$$\frac{3cxx}{bb+2bcx^3+ccx^6}=y. \qquad 1=v. \qquad \frac{1}{b+cx^3}=z.$$
$$\frac{-c}{bbxx+2bcx+cc}=y. \qquad 1=v. \qquad \frac{x}{bx+c}=z.$$
$$\frac{-2cx}{bbx^4+2bcxx+cc}=y. \qquad 1=v. \qquad \frac{xx}{bxx+c}=z.$$

In generall

$$\frac{ncx^{n-1}}{bb+2bcx^n+ccx^{2n}}=y. \qquad 1=v. \qquad \frac{1}{b+cx^n}=z.$$
$$\frac{b+2cx}{bbxx+2bcx^3+ccx^4}=y. \qquad 1=v. \qquad \frac{1}{bx+cxx}=z.$$
$$\frac{b+3cxx}{bbxx+2bcx^4+ccx^6}=y. \qquad 1=v. \qquad \frac{1}{bx+cx^3}=z.$$
$$\frac{2b+3cx}{bbx^3+2bcx^4+ccx^5}=y. \qquad 1=v. \qquad \frac{1}{bxx+cx^3}=z.$$

In generall

$$\frac{mbx^{m-1}+ncx^{n-1}}{bbx^{2m}+2bcx^{m+n}+ccx^{2n}}=y. \qquad 1=v. \qquad \frac{1}{bx^m+cx^n}=z.$$

~~cui~~ eodem modo

$\frac{-b+cxx}{bb+2bcxx+ccx^4}=y.$ $1=v.$ $\frac{x}{b+cxx}=z.$

$\frac{-bxx-2cx}{bbxx+2bcx+cc}=y.$ $1=v.$ $\frac{xx}{bx+c}=z.$

$\frac{-bx^4-3cxx}{bbx^4+2bcxx+cc}=y.$ $1=v.$ $\frac{x^3}{bxx+c}=z.$

$\frac{-2bx^3-3cxx}{bbxx+2bcx+cc}=y.$ $1=v.$ $\frac{x^3}{bx+c}=z.$

$\frac{-2bx^5-4cx^3}{bbx^4+2bcxx+cc}=y.$ $1=v.$ $\frac{x^4}{bxx+c}=z.$

$\frac{cd-eb}{dd+2edx+eexx}=y.$ $1=v.$ $\frac{b+cx}{d+ex}=z.$

$\frac{cd-2ebx}{dd+2edxx+eex^4}=y.$ $1=v.$ $\frac{b+cx}{d+exx}=z$

$\frac{2cd-2ebxx}{dd+2edxx+eex^4}=y.$ $1=v.$ $\frac{b+cxx}{d+exx}=z$

In generall

$$\frac{\overline{m-r}\times bdx^{m+r}+\overline{m-s}\times bex^{m+s}+\overline{n-r}\times cdx^{n+r}+\overline{n-s}\times cex^{n+s}}{ddx^{2r+1}+2edx^{r+s+1}+eex^{2s+1}}=y.\quad 1=v.\quad \frac{bx^m+cx^n}{dx^r+ex^s}=z.$$

$\frac{-3c}{2xx\sqrt{bx^4+cx}}=y.$ $1=v.$ $bx^3+c=z^2x^3.$

$\frac{-c}{xx\sqrt{bxx+c}}=y.$ $1=v.$ $bxx+c=z^2xx.$

$\frac{-c}{2x\sqrt{bxx+cx}}=y.$ $1=v.$ $bx+c=z^2x.$

$\frac{c}{2\sqrt{b+cx}}=y.$ $1=v.$ $b+cx=zz$

$\frac{cx}{\sqrt{b+cxx}}=y.$ $1=v.$ $b+cxx=zz$

$\frac{3cxx}{\sqrt{b+cx^3}}=y.$ $1=v.$ $b+cx^3=zz$

$\frac{bx^4-3c}{2xx\sqrt{bx^4+cx}}=y.$ $1=v.$ $bx^4+c=z^2x^3$

$\frac{bx^3-2c}{2xx\sqrt{bx^3+c}}=y.$ $1=v.$ $bx^3+c=z^2xx.$

$\frac{bxx-c}{2x\sqrt{bx^3+cx}}=y.$ $1=v.$ $bxx+c=z^2x.$

$\frac{b+2cx}{2\sqrt{bx+cxx}}=y.$ $1=v.$ $bx+cxx=zz.$

$\frac{b+3cxx}{2\sqrt{bx+cx^3}}=y.$ $1=v.$ $bx+cx^3=zz$

$\frac{b+4cx^3}{2\sqrt{bx+cx^4}}=y.$ $1=v.$ $bx+cx^4=zz$

$\frac{2bx^3-c}{2x\sqrt{bx^4+cx}}=y.$ $1=v.$ $bx^3+c=zzx.$

$\frac{2b+3cx}{2\sqrt{b+cx}}=y.$ $1=v.$ $bxx+cx^3=zz.$

$\frac{b+4cxx}{2\sqrt{b+cxx}}=y.$ $1=v.$ $bxx+cx^4=z^2.$

In generall

$\frac{mbax^{m}+nacx^{n}}{2x\sqrt{bx^m+cx^n}}=y.$ $a=v.$ $\sqrt{bx^m+cx^n}=z$

Also more generally.

$\frac{mabx^m+nacx^n+radx^r}{2x\sqrt{bx^m+cx^n+dx^r}}=v.$ $a=v.$ $\sqrt{bx^m+cx^n+dx^r}=z.$

$\frac{-ac}{bxx+c\sqrt{bxx+c}}=y.$ $a=v.$ $\frac{x}{\sqrt{bxx+c}}=z.$

$\frac{-ac}{2bx+2c\sqrt{bxx+cx}}=y.$ $a=v.$ $\frac{\sqrt{x}}{\sqrt{bx+c}}=z.$

$\frac{ac}{2b+2cx\sqrt{b+cx}}=y.$ $a=v.$ $\frac{1}{\sqrt{b+cx}}=z.$

$\frac{acx}{b+cxx\sqrt{b+cxx}}=y.$ $a=v.$ $\frac{1}{\sqrt{b+cxx}}=z.$

$\frac{ab+2acx}{2bx+2cxx\sqrt{bx+cxx}}=y.$ $a=v.$ $\frac{1}{\sqrt{bx+cxx}}=z.$

In generall

$\frac{mabx^{m-1}+nacx^{n-1}}{2bx^m+2cx^n\times\sqrt{bx^m+cx^n}}=y.$ $a=v.$ $\frac{1}{\sqrt{bx^m+cx^n}}=z.$

$\frac{3a}{2}\sqrt{b+cx}=y.$ $1=v.$ $\overline{b+cx}\sqrt{b+cx}=z.$

$3acx\sqrt{b+cxx}=y.$ $a=v.$ $\overline{b+cxx}\sqrt{b+cxx}=z.$

$\frac{9acxx}{2}\sqrt{b+cx^3}=y.$ $a=v.$ $\overline{b+cx^3}\times\sqrt{b+cx^3}=z.$

$\frac{-3ac}{2x^3}\sqrt{bxx+cx}=y.$ $a=v.$ $\overline{bx+c}\sqrt{bxx+cx}=zx.$

$\frac{-3ac}{x^4}\sqrt{bxx+c}=y.$ $a=v.$ $\overline{bxx+c}\sqrt{bxx+c}=zxxx.$

In generall

$\frac{3nacx^{n-1}}{2}\times\sqrt{b+cx^n}=y.$ $a=v.$ $\overline{b+cx^n}\times\sqrt{b+cx^n}=z.$

$\frac{-3ac\sqrt{bx^3+cx}}{x^3}=y.$ $a=v.$ $\frac{\overline{bx^2+c}\sqrt{bx^3+cx}}{x^5}=z.$

$\frac{3acx}{2}\sqrt{bx+cxx}=y.$ $a=v.$ $\overline{b+cx}\times\sqrt{bx+cxx}=zxx.$

$3acxx\sqrt{bx+cx^3}=y.$ $a=v.$ $\overline{b+cxx}\times\sqrt{bx+cx^3}=zxx.$

$\cancel{3acx^3\sqrt{}}$

$\frac{3acx^3}{2}\sqrt{b+cx}=y.$ $a=v.$ $\overline{b+cx}\times\sqrt{b+cx}=zx^3.$

$3acx^4\sqrt{b+cxx}=y.$ $a=v.$ $\overline{b+cxx}\times\sqrt{b+cxx}=zx^3.$

$\frac{3ac}{2}x^4\sqrt{bx+cxx}=y.$ $a=v.$ $\overline{b+cx}\times\sqrt{bx+cxx}=zx^5.$

$3acx^5\sqrt{bx+cx^3}=y.$ $a=v.$ $\overline{b+cxx}\sqrt{bx+cx^3}=zx^5.$

$\frac{3ac\sqrt{bx+cx^3}}{x}=y.$ $a=v.$ $\overline{bx+cx^3}\times\sqrt{bx+cx^3}=z.$

$\frac{-3ac}{2x^4}\times\sqrt{bx+c}=y.$ $a=v.$ $\overline{bx+c}\times\sqrt{bx+c}=z.$

$\frac{-3ac}{x^6}\sqrt{bx^3+cx}=y.$ $a=v.$ $\overline{bx^2+c}\times\sqrt{bx^3+cx}=zxx.$

$\frac{-3ac}{2x^6}\sqrt{bxx+cx}=y.$ $a=v.$ $\overline{bxx+cx}\times\sqrt{bxx+cx}=z.$

In generall

$\frac{3n-3m}{2}\times cx^{n-1}\times\sqrt{bx^m+cx^n}=y.$ $a=v.$ $\overline{bx^{2m}+cx^{n-3m}}\times\sqrt{bx^m+cx^n}=z.$

$\cancel{\frac{3ac\sqrt{}}{4x}}$

$\frac{2ba+5acx}{2}\sqrt{b+cx}=y.$ $a=v.$ $\overline{bx+cxx}\sqrt{b+cx}=z$

$\overline{ab+4acxx}\sqrt{b+cxx}=y.$ $a=v.$ $\overline{bx+cx^3}\times\sqrt{b+cxx}=z.$

$\frac{2ab+ac}{2xx}\times\sqrt{bxx+cx}=y.$ $a=v.$ $\overline{bx+c}\times\sqrt{bx^2+cx}=zx$

In generall

$\frac{\overline{3m+2r}\times bx^{m+r}+\overline{3n+2r}\times cx^{n+r}}{2x}\times a\sqrt{bx^m+cx^n}=y.$ $a=v.$

and $\overline{bx^{m+r}+cx^{n+r}}\times\sqrt{bx^m+cx^n}=z.$

And more generally,

$$\frac{\overline{2m+r}\times bdx^{m+r}+\overline{2m+s}\times bex^{m+s}+\overline{2n+r}\times cdx^{n+r}+\overline{2n+s}\times cex^{n+s}}{2x\sqrt{dx^r+ex^s}}\times a=y.$$

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$a=v.\quad \overline{bx^m+cx^n}\times\sqrt{dx^r+ex^s}=z.$

$\dfrac{3cdx}{2\sqrt{dx+e}}=y.\quad 1=v.\quad \overline{\dfrac{-2ce}{d}+cx}\times\sqrt{dx+e}=z.$

$\dfrac{3cdx^3}{\sqrt{dxx+e}}=y.\quad 1=v.\quad \overline{\dfrac{-2ce}{d}+cxx}\sqrt{dxx+e}=z.$

$\dfrac{-3cd}{2xx\sqrt{dx+exx}}=y.\quad 1=v.\quad \overline{\dfrac{-2cex}{d}+c}\times\sqrt{dx+exx}=zxx$

$\dfrac{-3cd}{x^4\sqrt{d+exx}}=y.\quad 1=v.\quad \overline{-\dfrac{2cexx}{d}+c}\times\sqrt{d+exx}=zx^3.$

~~$\dfrac{3cdx}{2\sqrt{dx+e}}=y.\quad 1=v.\quad \dfrac{-2ce}{d}+cx\times\sqrt{dx+e}=z.$~~

~~$\dfrac{3cdx^3}{\sqrt{dxx+e}}\quad 1=v.$~~

In generall

~~$\dfrac{-3cd}{2xx}$~~

$\dfrac{\overline{3m+3n}\times cdx^{3m+2n}}{2x\sqrt{dx^{3m+n}+ex^{2m}}}=y.\quad 1=v.\quad \overline{cx^n-\dfrac{2ce}{d}x^{-m}}\times\sqrt{dx^{3m+n}+ex^{2m}}=z$

$\overline{5ab+5bbx}\times\sqrt{a+bx}=y.\quad 1=v.\quad \overline{aa+2abx+bbxx}\sqrt{a+bx}=z.$

$\overline{5abx+\tfrac{5}{2}bbx^3}\sqrt{a+bxx}=y\cdot 1=v.\quad \overline{aa+2abx^2+bbx^4}\sqrt{a+bxx}=z.$

$\overline{5aax+10abxx+5bbx^3}\sqrt{ax+bxx}=y.\ 1=v.\ \overline{a^2x^2+2abx^3+bbx^4}\sqrt{ax+bxx}=z.$

$\dfrac{-5abx-\tfrac{5}{2}bb\sqrt{axx+bx}}{2x3}=y.\quad 1=v.\quad \overline{aaxx+2abx+bb}\sqrt{axx+bx}=zx^2.$

In generall

$\dfrac{5maax^{2m}+\overline{5m+5n}\times abx^{m+n}+5nbbx^{2n}}{2x}\sqrt{ax^m+bx^n}=y.\quad 1=v.$

And $\overline{aax^{2m}+2abx^{m+n}+bbx^{2n}}\times\sqrt{ax^m+bx^n}=z.$

$ax\sqrt{b+cx}=y.\quad \dfrac{a}{5c}=v.\quad \overline{6ccxx+2bcx-4bb}\times\sqrt{b+cx}=z.$

$\frac{+15a\varepsilon\varepsilon}{x^6\sqrt{\partial xx+\varepsilon}} = y$. $a = v$. $\overline{-3\varepsilon\varepsilon+4\partial\varepsilon x^2-8\partial^2 x^4}\sqrt{\partial xx+\varepsilon} = x^5 z\varepsilon$

$\frac{+15a\varepsilon\varepsilon}{2x^3\sqrt{\partial xx+\varepsilon x}} = y$. $a = v$ $\overline{-3\varepsilon\varepsilon+\varepsilon x-8\partial^2 xx}\sqrt{\partial xx+\varepsilon x} = x^3 z\varepsilon$

$\frac{+15a\varepsilon\varepsilon xx}{2\sqrt{\partial+\varepsilon x}} = y$. $a+v=0$. $-3\varepsilon\varepsilon x^2+4\partial\varepsilon x-8\partial\partial\sqrt{\partial+\varepsilon x} = z\varepsilon$

$\frac{+15a\varepsilon\varepsilon x^5}{\sqrt{\partial+\varepsilon xx}} = y$. $a+v=0$. $\overline{-3\varepsilon\varepsilon x^4+4\partial\varepsilon x^2-8\partial^2}\sqrt{\partial+\varepsilon xx} = z\varepsilon$

In Generall

$\frac{15na\varepsilon\varepsilon x^{3n-1}}{2\sqrt{\partial+\varepsilon x^n}} = y$. $a = v$ And

$\overline{3\varepsilon\varepsilon x^{2n}-4\varepsilon\partial x^n+8\partial\partial}\sqrt{\partial+\varepsilon x^n} = z\varepsilon$.

$15\partial\partial x\sqrt{\partial x+\varepsilon} = y$. $1 = v$. $\overline{6\partial\partial xx+2\partial\varepsilon x-4\varepsilon\varepsilon}\sqrt{\partial x+\varepsilon} = z$.

$60\partial\partial x^3\sqrt{\partial xx+\varepsilon} = y$. $1 = v$. $\overline{12\partial\partial x^4+4\partial\varepsilon xx-8\varepsilon\varepsilon}\sqrt{\partial xx+\varepsilon} = z$.

$\frac{-15\partial\partial}{x^4}\sqrt{\partial x+\varepsilon xx} = y$. $1 = v$. $\frac{\overline{-6\partial\partial-2\partial\varepsilon x+4\varepsilon\varepsilon xx}\sqrt{\partial x+\varepsilon xx}}{x^3} = z$

$\frac{-15\partial\partial\sqrt{\partial+\varepsilon xx}}{x^6} = y$. $1 = v$. $\frac{\overline{-3\partial\partial-\partial\varepsilon xx+2\varepsilon\varepsilon x^4}\sqrt{\partial+\varepsilon xx}}{x^5} = z$

In generall

$\frac{15n\partial\partial x^{2n}}{x}\sqrt{\partial x^n+\varepsilon} = y$. $1 = v$. $\overline{6\partial\partial x^{2n}+2\partial\varepsilon x^n-4\varepsilon\varepsilon}\sqrt{\partial x^n+\varepsilon} = z$.

$\frac{24\partial\partial xx-3\partial\varepsilon\varepsilon}{x}\sqrt{\partial xx+\varepsilon x} = y$. $1 = v$. $\left.\begin{array}{l}8\partial\partial xx\\ +2\partial\varepsilon x\\ -6\varepsilon\varepsilon\end{array}\right\}\times\sqrt{\partial xx+\varepsilon x} = z$.

$77\partial\partial x^4-5\partial\varepsilon\varepsilon\sqrt{\partial x+\frac{\varepsilon}{x}} = y$. $1 = v$. $\left.\begin{array}{l}14\partial\partial x^4\\ +4\partial\varepsilon xx\\ -10\varepsilon\varepsilon\end{array}\right\}\times\sqrt{\partial x^3+\varepsilon x} = z$

$\frac{8\partial\partial+\partial\varepsilon\varepsilon xx}{x^3}\sqrt{\partial+\varepsilon x} = y$. $1 = v$. $\overline{-4\partial\partial-2\partial\varepsilon x+2\varepsilon\varepsilon x^2}\sqrt{\partial+\varepsilon x} = xxz$.

$\frac{45\partial\partial+3\partial\varepsilon\varepsilon x^4\sqrt{\partial x+\varepsilon x^3}}{x^6} = y$. $1 = v$. $\left.\begin{array}{l}-10\partial\partial\\ -8\partial\varepsilon xx\\ +6\varepsilon\varepsilon x^4\end{array}\right\}\sqrt{\partial x+\varepsilon x^3} = x^5 z$.

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$\frac{32\partial\partial+4\partial\varepsilon\varepsilon x^4}{x^5}\sqrt{\partial+\varepsilon xx}=y.\quad 1=v.\quad \left.\begin{matrix}-8\partial\partial\\-4\partial\varepsilon xx\\+4\varepsilon\varepsilon x^4\end{matrix}\right\}\sqrt{\partial+\varepsilon xx}=x^4z.$

$-35\partial\partial xx-8\partial\varepsilon\varepsilon\sqrt{\partial x+\varepsilon}=y.\quad 1=v.\quad \left.\begin{matrix}10\partial\partial xx\\+2\partial\varepsilon x\\-6\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial x+\varepsilon}=z.$

$96\partial\partial x^4-12\partial\varepsilon\varepsilon\sqrt{\partial xx+\varepsilon}=y.\quad 1=v.\quad \left.\begin{matrix}16\partial\partial x^4+4\partial\varepsilon x^2\\-12\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial xx+\varepsilon}=z.$

$\frac{21\partial\partial+3\partial\varepsilon\varepsilon x^4}{x^5}\sqrt{\partial x+\varepsilon x^3}=y.\quad 1=v.\quad \left.\begin{matrix}-12\partial\partial\\-4\partial\varepsilon xx\\+2\varepsilon\varepsilon x^4\end{matrix}\right\}\sqrt{\partial x+\varepsilon x^3}=zx^4$

$48\partial\partial xx-15\partial\varepsilon\varepsilon\sqrt{\partial xx+\varepsilon x}=y.\quad 1=v.\quad \left.\begin{matrix}8\partial\partial xx\\+2\partial\varepsilon x\\-10\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial x^4+\varepsilon x^3}=z.$

$117\partial\partial x^4-21\partial\varepsilon\varepsilon\sqrt{\partial x^3+\varepsilon x}=y.\quad 1=v.\quad \left.\begin{matrix}18\partial\partial x^4+4\partial\varepsilon xx\\-14\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial x^5+\varepsilon x^3}=z.$

$\frac{12\partial\partial}{x^4}\sqrt{\partial+\varepsilon xx}=y.\quad v=1.\quad -4\partial\partial-4\partial\varepsilon xx\sqrt{\partial+\varepsilon xx}=zx^3$

$\frac{-\partial\partial-8\partial\varepsilon\varepsilon xx}{xx}\sqrt{\partial x+\varepsilon xx}=y.\quad 2\partial\partial-2\partial\varepsilon x-4\varepsilon\varepsilon x\sqrt{\partial x+\varepsilon x^2}=z.$

$63\partial\partial x^3-24\partial\varepsilon\varepsilon x\sqrt{\partial x+\varepsilon}=y.\quad v=1.\quad \left.\begin{matrix}14\partial\partial xx\\+2\partial\varepsilon x\\-12\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial x^5+\varepsilon x^4}=z.$

$80\partial\partial x^3-35\partial\varepsilon\varepsilon x\sqrt{\partial xx+\varepsilon x}=y.\quad v=1.\quad \left.\begin{matrix}16\partial\partial xx\\+2\partial\varepsilon x\\-14\varepsilon\varepsilon\end{matrix}\right\}\sqrt{\partial x^6+\varepsilon x^5}=z.$

$\frac{3\partial\partial-24\partial\varepsilon\varepsilon xx}{x}\sqrt{\partial x+\varepsilon xx}=y.\quad v=1.\quad 6\partial\partial-2\partial\varepsilon x-8\varepsilon\varepsilon x^2\sqrt{\partial x+\varepsilon xx}=z$

$99\partial\partial x^4-42\partial\varepsilon\varepsilon xx\sqrt{\partial x+\varepsilon}=y.\quad v=1.\quad \left.\begin{matrix}18\partial\partial x^5+2\partial\varepsilon x^4\\-16\varepsilon\varepsilon xxx\end{matrix}\right\}\sqrt{\partial x+\varepsilon}=z.$

$8\partial\partial-35\partial\varepsilon\varepsilon xx\sqrt{\partial+\varepsilon x}=y.\quad v=1.\quad 8\partial\partial x-2\partial\varepsilon xx-10\varepsilon\varepsilon x\sqrt{\partial+\varepsilon x}=z.$

$120\partial\partial x^4-63\partial\varepsilon\varepsilon xx\sqrt{\partial xx+\varepsilon x}=y.\quad v=1.\quad 20\partial\partial x^5+6\partial\varepsilon x^4-18\varepsilon\varepsilon x^3\sqrt{xx+\varepsilon x}=z.$

$\frac{-4\partial\partial-32\partial\varepsilon\varepsilon x^4}{xx}\sqrt{\partial+\varepsilon xx}=y.\quad v=1.\quad 4\partial\partial-4\partial\varepsilon xx-16\varepsilon\varepsilon x^4\sqrt{\partial+\varepsilon xx}=zx.$

$\frac{24\partial\partial-3\partial\varepsilon\varepsilon xx\sqrt{\partial+\varepsilon x}}{x^4}=y.\quad v=1.\quad -8\partial\partial-2\partial\varepsilon x-6\varepsilon\varepsilon xx\sqrt{\partial+\varepsilon x}=zx^3.$

In Generall.

$\overline{mm+8mn+15nn} \times \partial\partial x^{2n-1} - \overline{mm-2mn} \times \varepsilon\varepsilon x^{-1}$ in $\sqrt{\partial x^{m+n} + \varepsilon x^{m}} = y.$

$1=v.$ &, $\overline{2m+6n} \times \partial\partial x^{2n} + 2n\partial\varepsilon x^{n} - \overline{2m-4n} \times \varepsilon\varepsilon$ in $\sqrt{\partial x^{m+n} + \varepsilon x^{m}} = z.$